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Spontaneous creation of inflationary universes and the cosmic landscape

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Abstract: We study some gravitational instanton solutions that offer a natural realization of the spontaneous creation of inflationary universes in the brane world context in string theory. Decoherence due to couplings of higher (perturbative) modes of the metric as well as matter fields modifies the Hartle-Hawking wavefunction for de Sitter space. Generalizing this new wavefunction to be used in string theory, we propose a principle in string theory that hopefully will lead us to the particular vacuum we live in, thus avoiding the anthropic principle. As an illustration of this idea, we give a phenomenological analysis of the probability of quantum tunneling to various stringy vacua. We find that the preferred tunneling is to an inflationary universe (like our early universe), not to a universe with a very small cosmological constant (i.e., like today's universe) and not to a 10-dimensional (or a higher dimensional supercritical) uncompactified de Sitter universe. Some solutions are interesting as they offer a cosmological mechanism for the stabilization of extra dimensions during the inflationary epoch.

Keywords: D-branes, Superstring Vacua, Models of Quantum Gravity.

Contents

1. Introduction

String theory is the only known candidate for a unified theory of fundamental physics. Recent understandings in string theory and its compactifications [1]–[5] have led to the realization that string theory has many vacua (here we include metastable vacua with lifetimes comparable to or larger than the age of our universe). The number of such vacua, if not infinite, may be as huge as 10^{100} or larger [6]–[12]. How we end up in the particular vacuum we are in, that is, the particular site in this vast cosmic landscape, is a very important but highly non-trivial question. One may simply give up on this question by invoking the "anthropic principle". More positively, one may take the optimistic view that there exists a principle which tells us why we end up in the particular string vacuum we are in. In this paper, we propose such a principle. A better understanding of string theory and gravity may eventually allow us to check the validity of the idea, or maybe to improve on it. In the meantime, we may use this proposal as a working hypothesis and a phenomenological tool. As a minimum, even if our specific proposal turns out to be not quite correct, we hope it convinces some readers that such a principle does exist, allowing us to bypass the "anthropic principle".

Since observational evidence [13] of an inflationary epoch [14, 15, 16] is very strong, we suggest that the selection of our particular vacuum state follows from the evolution of the inflationary epoch. That is, our particular vacuum site in the cosmic landscape must be at the end of a road that an inflationary universe will naturally follow. Any vacuum state that cannot be reached by (or connected to) an inflationary stage can be ignored in the search of candidate vacua. That is, the issue of the selection of our vacuum state becomes the question on the selection of an inflationary universe, or the selection of an original universe that eventually evolves to an inflationary universe, which then evolves to our universe today. Let us call this the "Selection of the Original Universe Principle" or SOUP for short. The landscape of inflationary states/universes should be much better under control, since the inflationary scale is rather close to the string scale. Here we propose that, by analyzing all known string vacua and string inflationary scenarios, one may be able to phenomenologically pin down SOUP, or at least discover some properties of such a principle, which may then help in the derivation of SOUP in string theory. The key tool we shall use here is a modification of the Hartle-Hawking wavefunction [17].

Even if we understand inflation completely, its density perturbations, and all aspects of astrophysics related to galaxy and star formations, no one should expect us to be able to calculate from first principle the masses of our sun and our earth and why our moon has its particular mass. On the other hand, we do understand why the mass of our sun is not much bigger/smaller than its measured value. That is, our planetary system is a typical system that is expected based on our present knowledge. As theorists, we are comfortable with this situation. Along this line of thinking, one should feel content if the SOUP can show that our universe is among a generic set of preferred vacua, even if one fails to show why we must inevitably end in the precise vacuum state we are in. It is along this outwardly less ambitious, but probably ultimately more scientifically justified, direction that we are trying to reach.

Modern cosmology aims to describe how our universe has evolved to its present state from a certain initial state. An appealing scenario, due to Vilenkin [18, 19], Hartle and Hawking [17] and others [20], proposes that the inflationary universe was created by the quantum tunneling from "nothing", that is, a state of no classical space-time. This quantum tunneling from nothing, or the ultimate free lunch, should be dictated by the laws of physics, and it avoids the singularity problem that would have appeared if one naively extrapolates the big bang epoch backwards in time. It is natural to extend this approach to the brane

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world scenario, to see how brane inflation [21]–[26], natural in superstring theory, may emerge. Our understanding of superstring theory has advanced considerably in recent years so that it is meaningful to address this question. In this paper, we would like to consider some simple models motivated by superstring theory, to see what new features and issues may arise. We consider this as a first step towards the goal of understanding what string theory is trying to tell us about the origin of our Universe.

For SOUP in this free lunch, we need to determine the relative probability amplitude of tunneling from nothing to a variety of universes. This then allows us to select the universe with the largest probability amplitude. Suppose that the probability amplitude of this tunneling to a de Sitter universe is given by the Hartle-Hawking wavefunction of the universe with a cosmological constant Λ, in terms of the euclidean action S_E as

$$\Psi_{HH} \sim e^{-S_E} = e^{3\pi/2G_N\Lambda}\,. \tag{1.1}$$

Note that S_E is unbounded from below [27] and small Λ is exponentially preferred. Assuming the dark energy observed today is due to a very small cosmological constant $\Lambda_{\rm today}$, then it is exponentially more likely to tunnel directly to today's universe (or our universe many billions years in the future) than to an inflationary universe 13.7 billions years ago. If there exists a solution with an even smaller cosmological constant, then that universe will be exponentially more preferred. Clearly this is worse than the naive anthropic principle, and the above formula must be modified. In fact, it was pointed out that the above probability amplitude is unstable to corrections [28]. Intuitively, tunneling directly to today's universe with its immense size ($a \sim \Lambda_{\rm today}^{-1/2}$) must be suppressed.

To tunnel from nothing to an inflationary universe that describes our early universe, we need a reason that selects a tunneling to some intermediate value of $\Lambda_{\rm today} \ll \Lambda \ll G_N^{-1}$. (The observational data suggests that the tunneling to an approximate de Sitter universe with a Λ relatively close to the GUT scale is preferred.) As a phenomenological ansatz, we need to find an improved wavefunction by modifying the Hartle-Hawking wavefunction. Physically, we see that the above Ψ_{HH} has not included the effects of matter fields and the gravitational perturbative modes around the de Sitter metric. Naively, one may think that those effects are small. Here we shall argue otherwise.

We shall present 3 different (though ultimately equivalent) arguments for the necessity of such a modification: (i) destructive interference due to small fluctuations of large phases, (ii) quantum decoherence and (iii) space-like brane in string theory. The first argument is mostly intuitive, while the last two suggest the specific way the wavefunction should be improved.

Our first argument goes as follows: (1) In tunneling to a string vacuum state which then evolves classically involves first euclidean and then lorentzian time. Treating time as a coordinate, this means complex metric (more precisely, complex lapse function) is involved. This implies that, in the sum over paths in the evaluation of the wavefunction, we should include paths with complex metric, not just real metric. This was suggested by Halliwell and Louko [29] and others for 4-dimensional gravity. (2) In summing over paths, the steepest descent method is employed. This is standard practice. Here, we note that there is a large degeneracy in paths. That is, very different paths yields the same action with an imaginary part, $S = S_R + iS_I$. We expect this degeneracy to be lifted by the presence of gravitational and matter modes interacting with the classical metric. (3) When the phase S_I is very large (i.e., exponentially large compared to π) and S fluctuates, the path degeneracy is lifted and the sum over paths will in general lead to destructive interference, so that quantum effects (quantum tunneling here) will be suppressed. This is analogous to the situation of a macroscopic particle (say a marble or a billiard ball) in quantum mechanics. Here we make the assertion that this decoherence takes place at the lifting of the above path degeneracy with very large S_Is. We find that a large S_I phase appears generically in the quantum tunneling to a vacuum state with

(a) a large inflating volume, i.e., de Sitter size,

(b) a small cosmological constant Λ, and/or

(c) a large compactified extra dimensions.

(4) In particular, individual paths in the 4-dimensional de Sitter case contribute to Ψ an imaginary phase $\sim 1/G_N\Lambda$. So we expect destructive interference to suppress the tunneling probability in any of these situations. In particular, the resulting effect following from the sum over paths will suppress the tunneling to a small Λ universe. This result is very different from that suggested by eq. (1.1). However, a direct sum over such paths may be difficult. Usually, a rotation to euclidean space allows one to sum over all paths and evaluate such an effect. This will lead to a new term in the wavefunction. So we are led to propose the following modification to the Hartle-Hawking wavefunction;

$$\Psi \sim e^{\mathcal{F}}, \qquad \mathcal{F} = -S_E - \mathcal{D} \tag{1.2}$$

where the decoherence term $\mathcal{D}$ is real positive. This wavefunction reduces to the Hartle-Hawking wavefunction for $\mathcal{D} = 0$. For SOUP, $\mathcal{D}$ should be large in any of the three situations listed above; that is, the larger is the universe or the smaller is the cosmological constant, the larger is the value of $\mathcal{D}$, so their tunneling is suppressed.

The above argument can be made more precise in quantum decoherence. In decoherence, the classical metric (say, the cosmic scale factor a) is treated as the configuration variable while the perturbative modes around this metric and matter fields that couple to it are treated as the environment. The presence of the environment causes the quantum system to experience a dissipative dynamics, and the loss of quantum coherence results in the modification of the Hartle-Hawking wavefunction. In fact, the form of the leading term in $\mathcal{D}$ can be easily found. A simple generalization of the known results [30, 31, 32] (which were used to justify the classical treatment of time) to the tunneling case in string theory yields

$$\mathcal{D} = cV = c\left(\frac{M_s}{2\pi}\right)^9 V_9 + \cdots . \tag{1.3}$$

Here, c is a dimensionless constant and $M_s = 1/\sqrt{\alpha'}$ is the superstring scale. Here V is the dimensionless "spatial volume" (measured in $l_s \equiv 2\pi/M_s$) of the de Sitter (or any other) instanton, or the "area of the boundary" towards the end of tunneling. This is crudely the

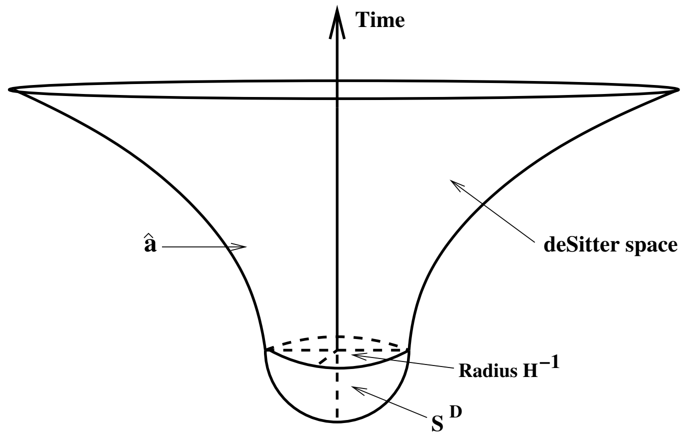

Figure 1: Starting with nothing, quantum tunneling happens via a S^D instanton with radius $a = 1/H$ to a D-dimensional de Sitter universe, which then grows to $\hat{a}$ and beyond.

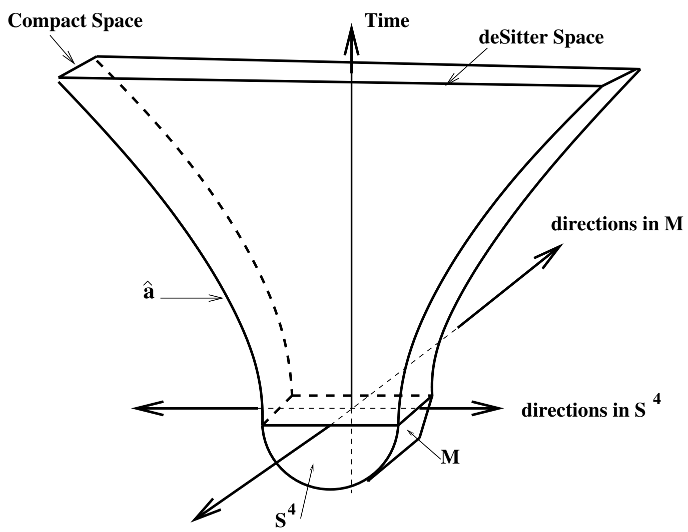

Figure 2: A $S^4 \times M$ instanton tunneling to the 4-dimensional de Sitter universe with a cosmologically stable 6 spatial dimensional space M. Some examples are $M = S^6, S^2 \times S^2 \times S^2$, three-fold Calabi-Yau manifold with fluxes etc.

transition region from euclidean to lorentzian space (see figures 1 and 2)). The origin of this term may be argued on physical grounds. Each mode of the environment supplies a mode-independent suppression factor so the resulting suppression factor is proportional to the total number of modes. Modes with wavelength longer than some fixed scale are unobservable and so should be traced over in the density matrix to yield the reduced density matrix. This cut-off implies that the total number of modes traced over is proportional to V_9, thus yielding the above $\mathcal{D}$ term. A constant term in $\mathcal{D}$ may be absorbed into the normalization of Ψ. Quantum contributions of matter fields may also contribute to the prefactor $\mathcal{P}$ in $\Psi = \mathcal{P} \exp(\mathcal{F})$.

Our third argument is indirect. In principle, $\mathcal{D}$ should be calculable in string theory, that is, the $\mathcal{D}$ term must have a calculable value for each of the potential string vacua, stable, metastable, or unstable. We may describe the tunneling as due to the presence of a S-brane. Since we are dealing with complex time (or euclidean to lorentzian time), we call such a space-like brane a S'-brane. A boundary term involving a S'-brane is proportional to V_9, leading us to the same term in $\mathcal{D}$ (1.3).

The V_9 term suppresses universes with a small Λ and/or a large inflationary size. To suppress universes with very large (or uncompactified) extra dimensions, we need additional terms. (We do not expect $\mathcal{D}$ to be simple.) Possible simple terms are V_{10}^2, $V_9 V_{10}$ etc. In cases where the extra dimensions are dynamically stabilized (even only to a metastable state), we may use the cV term only (or a V_{10} term) to illustrate the approach. c may be a function of the vacuum state as well. It is important to determine the functional form of $\mathcal{D}$ in string theory in a more careful analysis. For our purpose here, the above choice of $\mathcal{D}$ is enough to explain our main points.

In string theory, both S_E and $\mathcal{D}$ (and so $\mathcal{F}$ and Ψ) should be calculable for each possible vacuum state. The inflationary vacuum state that ends in our today's universe should be the vacuum state that maximizes Ψ or $\mathcal{F}$. If this SOUP program is successful, we should be able to understand why we are where we are (why 3 large dimensions, why at most $N = 1$ supersymmetry, why $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ etc.), thus avoiding the anthropic principle.

As an illustration, we simply consider phenomenologically the above explicit term in $\mathcal{D}$ that will provide the suppressions we are looking for and can be applied to known string vacua. To illustrate the SOUP idea, let us apply the above specific ansatz to the brane inflationary scenario proposed by KKLMMT [22]. We use G_N and the COBE density perturbation data to crudely fix M_s, Λ and c, namely $M_s \simeq 4 \times 10^{17}$ GeV, $\sqrt{\Lambda} \sim 10^{14}$ GeV and $c \simeq 10^{-3}$. Maximizing $\mathcal{F}$ fixes the fluxes that stabilize the Calabi-Yau manifold. We find that $\mathcal{F} \simeq 10^{18}$. Using the determined value of c, we find the value of $\mathcal{F}$ for the tunneling directly to a KKLT vacuum [2] without inflation (which has $\mathcal{F} < 0$). On the other hand, tunneling to a ten-dimensional de Sitter universe S^{10}, or other similar universes such as $S^4 \times S^6$, $S^5 \times S^5$ etc. (with Λ suppied by $D9 - \bar{D}9$-brane pair) has $\mathcal{F} \simeq 10^9$, largely independent of the value of c. In summary, the inflationary universe is much preferred among the vacua we examined. This is not surprising, since the functional form of $\mathcal{F}$ for a D-dimensional inflationary universe (with the remaining compactified dimensions stabilized) is,

$$\mathcal{F} = \frac{a}{\Lambda^{(D-2)/2}} - \frac{b}{\Lambda^{(D-1)/2}} + \cdots . \tag{1.4}$$

As shown in figure 3, for generic constants a and b, an intermediate value of Λ is clearly preferred over a very large or a very small Λ. In particular, tunneling directly to a supersymmetric vacuum is severely suppressed. To see which inflationary scenario or some other vacuum state is preferred via tunneling, a much more precise determination of Ψ is required. Note that SOUP will not work if we use the alternative to the Hartle-Hawking wavefunction [33, 34], where the sign of the first term becomes negative. In this case, large Λ (easily achieved with many brane-antibrane pairs) is preferred, which leads to the breakdown of the WKB approximation.

Although the details of this preliminary analysis is admittedly quite simplistic, it does open the possibility that a good understanding of the decoherence term may select a

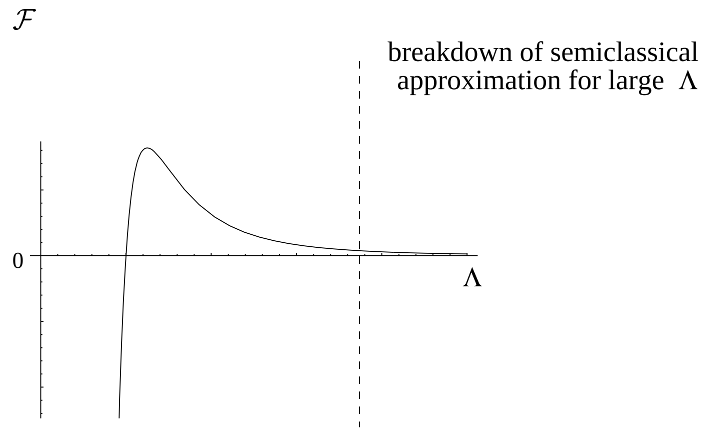

Figure 3: $\mathcal{F}$ as a function of Λ. The semi-classical approximation breaks down (indicated by the dashed vertical line at the right) when Λ is large.

particular inflationary state which will then evolve to our today's universe after inflation. A more detailed analysis may allow us to determine more completely the functional form of $\mathcal{D}$ (and the value of c). At the same time, as one examines more string theory vacua and inflationary solutions, one should be able to phenomenologically refine the functional form of $\mathcal{D}$ as well. This is the basic point of SOUP. The reader may think that SOUP is too good to believe. We argue that the anthropic principle is too hard to believe; almost any scientifically motivated alternative is more desirable.

Slow-roll inflation usually implies some form of eternal inflation [35, 36]. With eternal inflation, it is argued that the origin of the universe issue is less pressing. Following our view point, this does not explain why the particular eternal inflationary universe is selected in the first place. If eternal inflation has happened eternally, the anthropic principle must be invoked in this scenario to explain why the particular inflationary universe is selected. On the other hand, SOUP will select the particular inflationary universe, even if it has the eternal inflationary property. A somewhat similar comment may be applied to the appearance of inflationary (or some other) universes from a "meta-universe" [37, 38]. We must first understand the origin of such a meta-universe.

If successful, this program may also be used to make predictions that can be tested in the near future. In the brane inflationary scenario, cosmic strings are produced towards the end of inflation and they will leave signatures to be detected [39]–[46]. However, the type/property of the cosmic strings produced depends crucially on the particular brane inflationary scenario that had taken place. If SOUP can select the specific brane inflationary universe emerging from tunneling, it will give predictions on the type/property of the cosmic strings to be detected.

It was pointed out that supercritical ($D > 10$) string vacua [47, 48, 49] may also provide a realistic description of today's universe. However, supersymmetry is typically absent in such supercritical vacua, so low energy supersymmetry will not be detected in LHC and similar experiments. It is fair to ask if SOUP can distinguish between critical and supercritical string vacua. Although SOUP clearly prefers S^{10} over S^D for $D > 10$, with

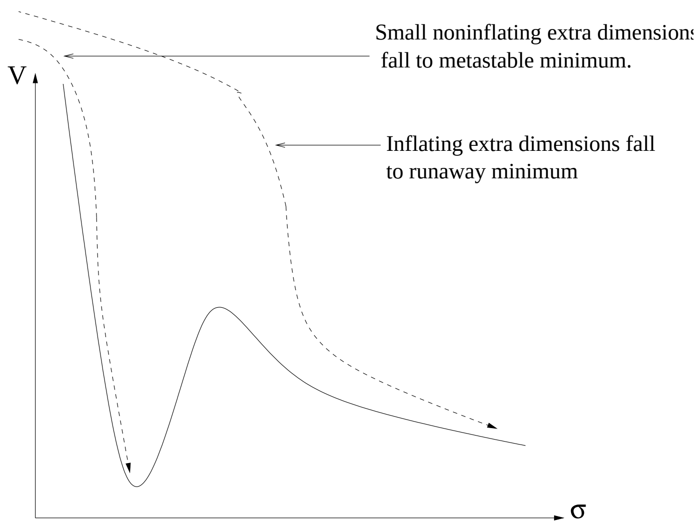

Figure 4: The potential $V(\sigma)$ as a function of the size σ of the compactified dimensions. If the size of the extra dimensions grows substantially in the early universe, the universe will end up with 10 uncompactified dimensions. So cosmological stabilization may be needed in addition to dynamical moduli stabilization.

$\mathcal{F}(D > 10) < 10^5$, it is not so clear for inflationary scenarios in critical versus supercritical string vacua, assuming viable supercritical inflationary scenarios can be realized. If so, the determination of the string coupling g_s among various inflationary scenario via the maximization of $\mathcal{F}$ may be able to distinguish them. Clearly a better understanding of $\mathcal{F}$ should be valuable as well.

In this speculative paper, the following issues are discussed. In section 2, we argue for the destructive interference in the path sum and propose the wavefunction that includes the decoherence term. We discuss the decoherence approach to find that the form of the leading term in $\mathcal{D}$ is the above V_9 term. In section 3, we apply this wavefunction for the quantum tunneling from nothing via a 10-dimensional de Sitter instanton S^{10} to a 10-dimensional de Sitter space. The cosmological constant comes from the pair creation of branes, the simplest being $D9$-$\bar{D}9$-brane pairs in type-IIB models or non-BPS $D9$-branes in type-IIA models. In this paper, we shall focus on type-IIB or F theory vacua. We also consider other cases, such as $S^4 \times S^6$, $S^5 \times S^5$, $S^4 \times S^2 \times S^2 \times S^2$ $S^3 \times S^3 \times S^4$, $S^4 \times T^1 S^2 \times T^3$ etc.. Quantum tunneling happens at a scale below the superstring scale, so that semi-classical gravitational analysis should be valid. Among this set of vacua, we find that, independent of the value of $\hat{c}$, the symmetric S^{10} de Sitter universe has the largest tunneling probability, with $\mathcal{F} \simeq 10^9$. If the tunneling is via a $S^4 \times M$ with a pair of $D9$-brane-$\bar{D}9$-brane, only the 3 spatial dimensions in S^4 will grow exponentially (inflate), while the other dimensions are cosmologically stabilized, i.e., time-independent (though the amount of inflation is negligible due to the fast tachyon-rolling). Introducing the dilaton does not change these qualitative features.

Even if we assume that the moduli are all stabilized in today's universe, there is no guarantee that they should be stabilized during the inflationary epoch. (See figure 4.) In

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KKLT, the Kähler moduli are only metastabilized; that is, if they start out with relatively large values, they will continue to grow and the universe decompactifies to a higher dimensional universe. (In the simple case of a single Kähler modulus, the universe decompactifies to 10 dimensions.) This runaway solution is always present in superstring theory. So, if the extra dimensions grow during inflation, they may easily grow by a big enough factor and decompactification takes place. The presence of cosmological stabilization of the extra dimensions ensures that the Kähler moduli are not growing, so towards the end of inflation, they can fall down to the locally stabilized values, as indicated in figure 4. In our scenario, universes with large sizes of the extra dimensions are suppressed by the decoherence term. After tunneling, they remain unchanged during inflation. Their stabilization can happen before, during or immediately after inflation.

We then consider the brane inflationary scenario [21] as realized in string theory by Kachru et. al. [22]. When we apply the wavefunction Ψ to this scenario, it actually selects a specific inflationary scenario with the fluxes determined. That is, after inflation, the universe settles down to a specific vacuum state, where all the fluxes (say (M_1, K_1)) and the moduli are fixed. One may consider tunneling directly to such a KKLT vacuum state without going through inflationary stage. Such a preferred vacuum state will have different fluxes (say (M_0, K_0)). We find that tunneling to such a vacuum state without going through inflation is suppressed relative to the tunneling to an inflationary vacuum state which then ends up in the (M_1, K_1) state. In the search of possible vacuum state for us to live in, this implies that the (M_0, K_0) vacuum state is not preferred. Of course, subsequent rolling [50] and tunneling from the (M_1, K_1) state to another (M_2, K_2) state are still possible [51, 52]. After a discussion on supercritical string vacua, we conclude with some remarks.

Our argument for the decoherence term is admittedly crude and naive. However, further refinement of the wavefunction Ψ is likely to retain some of these qualitative properties. Examination of more possible string vacua will help us to pin down the properties of Ψ. This approach offers the hope that SOUP will select a particular inflationary universe that evolves to a specific vacuum and thus the anthropic principle may be avoided.

Many of the details are relegated to the appendices.

2. Wavefunction of the universe and tunneling probability

The qualitative picture of quantum tunneling from nothing to an inflationary universe is relatively well accepted. However, the correct wavefunction of the universe, or equivalently, the formula for the tunneling probability, has remained controversial. Here we give a brief introduction (see appendix A for a sketchy review). We then present our proposal on the improved wavefunction and the corresponding tunneling probability. We first discuss the issues for a de Sitter instanton in 4 dimensions. We then generalize the discussion to 10 dimensions. Finally, we propose a phenomenological realization of the decoherence effect to be incorporated into the wavefunction.

2.1 Birth of de Sitter universes

Let us first review the creation of an inflationary universe in 4 dimensions. (The gener-

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alizations to other dimensions are straightforward and will be discussed later.) We start with the action for a closed universe M,

$$S = \frac{1}{16\pi G}\int_M d^4x\sqrt{|g|}\,(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial M} d^3x\sqrt{|h|}K + \int_M d^4x\sqrt{|g|}L_m(\phi) \tag{2.1}$$

where $\Lambda \geq 0$ is the cosmological constant and L_m is the lagrangian for all matter fields labeled as ϕs. Here h_{ij} is the 3-metric on the boundary ∂M and K is the trace of the second fundamental form of the boundary. The extrinsic curvature K in the York-Gibbons-Hawking surface term will be important for our discussion. Here the universe is assumed to be closed, homogeneous and isotropic. To get the picture of the tunneling, consider figure 1, with metric $ds^2 = N(\tau)^2 d\tau^2 + a(\tau)^2 d\Omega_3^2$, where the scale factor $a(t)$ in the de Sitter region with lapse function $N = -i$ (where $t = N\tau = -i\tau$) is the solution of the Einstein evolution equation:

$$\dot{a}^2 + 1 = H^2 a^2\,, \tag{2.2}$$

where H is the Hubble parameter, $H^2 = \Lambda/3$. This describes the de Sitter space:

$$a(t) = \frac{1}{H}\cosh(Ht) \tag{2.3}$$

in the absence of matter field excitations. For positive t, this describes the inflationary universe. The euclidean version of the action (2.1) can be obtained by replacing $t \to -i\tau$. This gives the euclidean version of eq. (2.2):

$$-\dot{a}^2 + 1 = H^2 a^2 \tag{2.4}$$

which gives the S^4 solution:

$$a(\tau) = \frac{1}{H}\cos(H\tau)\,. \tag{2.5}$$

This is the well-known de Sitter instanton, which is a compact space. It is half a four-sphere and is defined only for $|\tau| \leq \pi/2H$. This instanton is interpreted as the tunneling from nothing (i.e., no classical space-time) to an inflationary universe at $\tau = 0$ with size $a = 1/H$ and $\dot{a} = 0$, as shown in figure 1. We expect the vacuum energy to come from an inflationary potential $V(\phi)$, with its (local) maximum at Λ.

The curvature of the S^4 instanton solution is $R = 12H^2$. For $G_N\Lambda \ll 1$, semiclassical approximations should be valid and we may calculate the tunneling probability following standard field theory procedures: $P_{HH} \simeq e^{-S_E}$ where S_E is the euclidean action of half a S^4 instanton. However,

$$S_E = -\frac{3\pi}{2G_N\Lambda} \tag{2.6}$$

which is negative. (The entropy $S_{entropy} = -S_E$.) In fact, the euclidean action for gravity is unbounded from below. This requires a closer look at the tunneling probability.

Following Hawking [53], let us start with the wavefunction of the universe Ψ in the path integral formalism. For spatially closed universes, one may express $\Psi[h_{ij}]$ as

$$\Psi[h_{ij}] = \int_{\emptyset}^{h_{ij}} Dg_{\mu\nu}e^{-S[g_{\mu\nu}]} \tag{2.7}$$

where h_{ij} is the 3-metric and matter fields are ignored for the moment to simplify the discussion. Here S is the classical euclidean action. The functional integral is over all 4-geometries with a space-like boundary on which the induced metric is h_{ij} and which to the past of that surface there is nothing (see figure 1).

Consider the tunneling from nothing to half a four-sphere ($\tau = -\pi/2H$ to $\tau_0 = 0$ with $a = 1/H$) and then the universe evolves classically in the inflationary epoch ($\tau > \tau_0$ to $\hat{a} > 1/H$, as indicated in figure 1). The action has the value (see appendix B):

$$S(\hat{a}) = S_R + iS_I = -\frac{3\pi}{2\Lambda}\left(1 \mp i((H^2\hat{a}^2 - 1)^{\frac{3}{2}}\right). \tag{2.8}$$

We may view the real part S_R as the euclidean action due to tunneling while the imaginary part S_I as the phase change due to the classical evolution of the inflationary universe.

It is more convenient to let τ be a parameter or coordinate ($0 \leq \tau \leq 1$), so the transition from euclidean to lorentzian time is facilitated by the transition of the lapse function from real ($N = 1$) to pure imaginary ($N = -i$) values. Once we are ready to entertain complex N, we should include the tunneling from nothing directly to an universe with size $\hat{a}$, where $H\hat{a} > 1$. In this case, the path integral is

$$\Psi(a) = \int_C dwdz\mu(z, a, w)\exp(-S(z, a, w)) \tag{2.9}$$

where w symbolically labels the different paths in the complex time T-plane, and z parametrizes the instanton (see appendix B for details). It turns out that $S(z, a, w) = S(z, a)$. Using the steepest-descent method, where the steepest-descent paths are the paths with S_I =constant, we find again the above action (2.8), where the saddle points correspond to $H\hat{a} > 1$.

Next we consider the ten dimensional case, which corresponds closer to the situation in string theory. The analysis for S^{10} is very similar to that of the above S^4 case. Other cases such as the $S^4 \times S^6$ (or more generally $S^{1+n_1} \times S^{n_2}$) instanton does not differ much from the S^4 case; although the S^4 component becomes the de Sitter space, the size of the S^6 is fixed, with its value dictated by the same Λ (see next section and appendix E for more discussions). The case of $S^4 \times M$ where the compactification of M is dynamically stabilized is also quite similar to the above case. The case that is interestingly new (from our perspective) is when the size of M is not fixed. One may consider either the case where the size of M also varies during inflation (e.g., Kasner-like solutions), or the case where the size of M is static during inflation. In either case, the result we are looking for does not differ much. Since the latter case is more novel, we shall focus on the $S^4 \times M$ case where M is cosmologically but not dynamically stabilized. To be specific, let us consider the case of $S^{1+n_1} \times T^1 S^{n_2} \times T^m$ where T^1 is fibered over S^{n_2}. The Einstein equation is easy to solve. See details in appendix E. To match onto the earlier discussion, let us consider $S^4 \times T^1 S^2 \times T^3$. We find that, after tunneling, S^4 turns into de Sitter space with exponentially growing $a(t)$, while both the S^2 radius and the torus radii are constant (see figure 4). That is, they are cosmologically stabilized during the inflationary epoch. Although the S^2 radius is fixed in terms of Λ, the torus radii are undetermined. If they

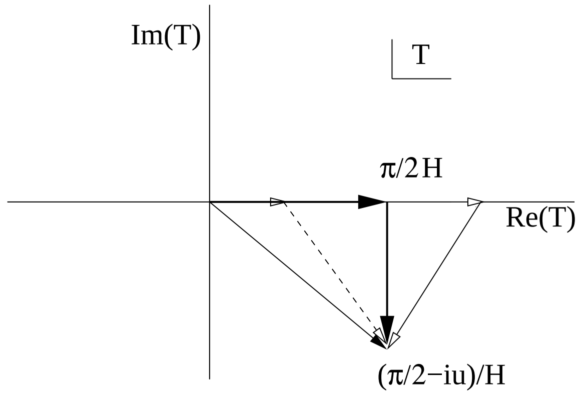

Figure 5: The 4 paths for $T = 0$ to $T = (\pi/2 - iu)/H$ discussed in the text, from left to right: $w = 0$, $0 < w < \pi/2H$, $w = \pi/2H$ and $w > \pi/2H$. Actually, there are infinitely many paths reachable simply by analytic continuation. Here $\cosh u = \hat{a}H$.

start out too big, after inflation they will enter the regime shown in figure 4 where they will simply grow and become decompactified.

Following a similar analysis for the above S^4 case, we find that, for the 10-dimensional vacuum and $H\hat{a} > 1$, after summing over all degenerate paths:

$$S(\hat{a}) = S_E + iS_I = -\frac{90\pi V_6}{42G_{10}\Lambda}\left(1 \mp i((H\hat{a})^2 - 1)^{\frac{3}{2}}\right) \tag{2.10}$$

where G_{10} is the ten-dimensional gravitational constant, V_6 is the compactification volume of $T^1S^2\times T^3$, $H^2 = 7\Lambda/30$ and $\hat{a}$ is the cosmic scale factor of the de Sitter space immediately after tunneling. We see that the imaginary part S_I of $S(\hat{a})$ is generically large if (i) Λ is small, (ii) $\hat{a}$ is large, and/or (iii) V_6 is large. Generalization to other cases is straightforward. For a D-dimensional inflationary universe with n dimensional compactified volume V_n, one has, for $\hat{a} \gtrsim 1/H$,

$$S(\hat{a}) \propto -\frac{\hat{a}^{D/2}V_n\Lambda}{G_{D+n}}\left(1 \mp i((H\hat{a})^2 - 1)^{\frac{3}{2}}\right). \tag{2.11}$$

2.2 Lifting the Feynman path degeneracy

Following the decoherence criterion, tunneling in each of the above 3 situations should be suppressed when the the effect of matter fields is included. Note that the above formula seems to suggest that, for $H\hat{a} = 1$, $S_I = 0$ even for large V_6 and $1/\Lambda$. It is our assertion that the lifting of the path degeneracy takes place in a way such that generic S_I is small only if each of the factor is not big. This may be a very strong assumption.

Introducing $T = N\tau$, the integral over T in the action involves an analytic function in which the contour in the complex T-plane can take a variety of paths, a subset of which is shown in figure 5. It is convenient to shift the τ coordinate $\tau \to \tau + \pi/2H$, so τ is always positive. For fixed z, let us consider 4 paths (among infinitely many) shown in figure 5 to illustrate some of the points we would like to make.

- $w = \pi/2H$. This integration of T along the real axis (i.e., euclidean time: τ from 0 to 1 with $N = \pi/2H$) corresponds to tunneling to half a four-sphere, and then the de Sitter universe evolves classically as T goes in the imaginary direction (i.e., lorentzian time) from $\pi/2H$ to $(\pi/2 \mp iu)/H$, where $\cosh u = \hat{a}H$. This is the only path included in the Hartle-Hawking wavefunction.

- $w = 0$. The integration is along the $T = N\tau$ path for

$$N = \frac{\left[\frac{\pi}{2} - iu\right]}{H}$$

with τ going from 0 to 1.

- $0 < w < \pi/2H$. This corresponds to tunneling to less than a half four-sphere (with $Ha < 1$), and then continue tunneling to $\hat{a}$.

- $w > \pi/2H$. This corresponds to tunneling to larger than a half instanton (with $Ha < 1$), and then continue tunneling back to $\hat{a}$.

These 4 paths (and many others related by analytic continuation in the complex T-plane) all have the same action given by $S(z, a, w) = S(z, a)$ (B.9). The reason they have the same action is because of the gauge symmetry coming from diffeomorphism invariance [81]. Ignoring the measure and summing over paths via the steepest descent approximation discussed earlier and in appendix B then yields the result (2.8) or (2.10) as given above. Let us make a few observations here:

1. We are interested in an inflationary universe with growing a. Even if we tunnel to an inflationary universe with $Ha < 1$, a will rapidly grow to $\hat{a}$ such that $H\hat{a} > 1$. Summing over all paths that end at $\hat{a}$, the real part of Ψ goes like $-3\pi/2G\Lambda$ in (2.8) or $\sim V_6/G_{10}\Lambda$ in (2.10), corresponding to tunneling to exactly half an instanton. This seems to uniquely fix the tunneling probability. This answer is same as that adopted by Hartle and Hawking [17] in the 4-dimensional case, though the argument is very different, since they do not consider paths involving complex metric.

2. So far, the measure and the matter fields have been ignored.

$$S_{\text{eff}}(z, a, w) = S(z, a) + S_m(z, a, \phi) - \ln \mu(z, a, w)\,. \tag{2.12}$$

Besides the the measure, which receives contributions from gauge fixing as well as other effects, S_m includes light matter (both bosonic and fermionic) fields as well as excitational modes arising from perturbing the metric. These modes/fields are collectively represented by ϕ. Intuitively, we expect the inclusion of these fields to lift the above path degeneracy. This will lead to an effective action $S_{\text{eff}}(z, a, w)$ that is path-dependent. Its dependency on w will lead to important effects when the imaginary part S_I of the action is large. A particularly interesting field to include is the inflaton field in a slow-roll inflationary scenario. In this case, Λ is actually the effective inflaton potential $V(\phi)$ and the de Sitter description is only approximate. Following the $w = \pi/2H$ path, scale-invariant density perturbation is generated during the classical evolution of the universe. For other paths, say the $w = 0$ path, presumably no ϕ fluctuation is produced during tunneling. In any case, whatever density perturbation generated during tunneling is expected to be very different from that generated during inflation. This illustrates the lifting of the path degeneracy.

3. Suppose $Im\ S(\hat{a}, w)$ is very large; a small lifting of the path degeneracy due to S_m (and $\mu(z, a, w)$) will introduce a perturbative change in $S(z, \hat{a}, w)$, which can still be much larger than π. This destructive interference among the paths is akin to decoherence, and any quantum effect will be suppressed, leaving behind only classical behavior. Since classically there is no tunneling, this will suppress the tunneling probability. Let us assume that decoherence takes place when $Im\ S(\hat{a})$ is very large. Generically this happens in the following 3 situations:

   - Very small Λ. A phase variation due to S_m will suppress the quantum tunneling. That is, in a model where Λ is dynamical, the quantum tunneling to a de Sitter universe with a very small cosmological constant is suppressed. With today's dark energy, $(G\Lambda)^{-1} \sim 10^{120}$. So a very small lifting of the path degeneracy that causes a tiny percentage change in the value on S will be sufficient to suppress the tunneling to a de Sitter universe with a very small cosmological constant.
   - $H\hat{a} \gg 1$. The lifting of path degeneracy will suppress the quantum paths that tunnel directly or close to a large $\hat{a}$. This suggests that tunneling from nothing to a de Sitter universe with a not too large (say, close to but a little larger than H^{-1}) is most likely. The universe then evolves classically (i.e., inflates) from such a value of a to $\hat{a}$ and beyond. This allows us to preserve the almost scale-invariant density perturbation generated during inflation.
   - Large V_6. Again, a phase variation due to S_m will suppress its quantum tunneling. That is, in a model where size of the extra dimension is dynamical, the quantum tunneling to a de Sitter universe with a very large compactification volume (or in the uncompactified limit) is suppressed.

4. Since the real part of the action S_R suppresses tunneling to a vacuum with a very large Λ, we expect the desirable value of Λ to take some intermediate value below the Planck scale. This then justifies the semi-classical approximation we have been relying on.

2.3 An improved wavefunction

In usual quantum mechanics, decoherence occurs when a macroscopic object interacts with its environment, e.g., a dust particle in the bath of the cosmic microwave background radiation in outer space. However, for the universe, there is no outside observer to play the role of the environment. Instead, the classical curved metric plays the role of the macroscopic object, while the perturbative modes of gravity as well as the matter fields interacting with classical gravity play the role of the environment, as explained by Kiefer and others [30, 31]. Since this point is rather important in SOUP, we give a brief review in appendix C.

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The S_E term in Ψ suppresses the tunneling to a large Λ universe. For small Λ, the universe is large, i.e., macroscopic, so we expect decoherence to suppress its tunneling (since there is no tunneling in classical physics). Phenomenologically, we expect an inflationary universe with a moderate value of Λ (which is much larger than today's dark energy and smaller than the Planck scale) that evolves more or less classically before it ends with the hot big bang epoch. In de Sitter space or an inflationary universe, the cosmic scale size $\hat{a}$ plays the role of the configuration variable. Here we would like to find the leading term in $\mathcal{D}$ due to the environment.

A mildly path-dependent measure $\mu(z, a, w)$ and $S_m(\phi)$ may allow us to realize this scenario via decoherence. For what values of Λ that this decoherence effect will begin to suppress the tunneling is a model-dependent question. The standard way to control a path integral that involves phases is to go to imaginary time. Such a calculation is beyond the scope of this paper. Instead, let us take a phenomenological approach here to find the functional form of the leading contribution to $\mathcal{D}$. Let us consider two approaches: S'-brane and decoherence. They yield the same functional form for the leading term in $\mathcal{D}$.

- Although all paths in the Feynman path integral approach involve continuous metric, paths with discontinuous derivatives of the metric are allowed; that is, the extrinsic curvature K can have jumps so that its integral in the action can be finite. The discontinuity of K corresponds to a time-localized defect. In string theory, such "defects" are known as space-like branes or S-branes [54]. Since the defect here is located at complex time, we shall call it S'-brane. More generally, a S'-brane has some structure, which is reflected in a rapidly changing (but smooth) K in some time region. That is, at the location of such a S'-brane, the boundary term can be expressed as

$$\frac{1}{16\pi G}\int_{\partial M} d^9x\sqrt{|h|}K = +T_{S'}\int d^9x\sqrt{|h|} \simeq T_{S'}V_9 \tag{2.13}$$

where $T_{S'}$ indicates the S'-brane tension and V_9 the 9-spatial volume. For the $w = \pi/2H$ case, V_9 is simply the "boundary area" of the half-ten-sphere. One anticipates that the tension of such a S'-brane to be around the string scale (up to factors of 2π). So we are led to the following ansatz:

$$\begin{aligned}\Psi &\simeq \exp\mathcal{F} = \exp\left(-S_E - \mathcal{D}\right)\\ \mathcal{D} &\simeq cV + \cdots = cV_9/l_s^9 + \cdots\,.\end{aligned} \tag{2.14}$$

where c is a dimensionless parameter, V is the dimensionless spatial volume, $l_s = 2\pi\sqrt{\alpha'} = 2\pi/M_s$, and V_9 is the "area of the boundary" at the transition from euclidean to lorentzian space (see figures 1 and 2). That is, cV is the leading term in the decoherence term $\mathcal{D}$. Here, c may have a mild dependence on the matter fields, both open and closed string modes, as well as on the excitational modes emerging from perturbing the classical metric. The determination of the value of c will be a challenge. Sometimes it is convenient to introduce $\hat{c}\hat{V}$ where $\hat{V}$ is the "spatial volume" of the de Sitter (or any other) instanton (as measured in string scale $M_s^2 = 1/\alpha'$); that is, $\hat{V} = M_s^9 V_9$ for critical string vacua. So $c = (2\pi)^9\hat{c}$.

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A constant term in $\mathcal{D}$ may be absorbed into the normalization of Ψ. The cV term suppresses universes with a small Λ and/or a large inflationary size. To suppress universes with very large (or uncompactified) extra dimensions, we need higher order terms. Possible terms like V_{10}^2, V_9V_{10} etc. can easily do the job, but terms like S_E^2, V_9^2 etc are not useful. Fortunately, a realistic string vacuum will have the compactification of its extra dimensions dynamically stabilized. In those realistic situations, we may assume the higher order terms in $\mathcal{D}$ to be negligible. So, with some luck, a phenomenological analysis keeping only the lowest order term in eq. (1.2) may be sufficient.

- Now we turn to the quantum decoherence approach [55, 30, 56], which gives the same leading term in $\mathcal{D}$. In quantum cosmology the total wavefunction in a Friedmann solution with cosmic scale a is

$$\Psi(a, x_n) = \psi_o(a) \prod_{n>0} \chi_n(a, x_n) \tag{2.15}$$

where $\psi_o(a)$ is (up to a normalization factor) the Hartle-Hawking wavefunction, and $\chi_n(a, x_n)$, with amplitudes x_n, (an orthonormal set at a given value of a) are the contributions from all other fields. Here the metric a is the configuration variable describing the semi-classical evolution of the universe while the higher multipoles of gravity and other matter fields interacting with gravity provides the environment. These fields interact with the gravitational contribution encoded in $\psi_o(a)$ and the effect of this interaction can be seen by considering the reduced density matrix, where the environment is traced over:

$$\rho_S(a; a') = \mathrm{tr}_{x_n}\left(|\Psi\rangle\langle\Psi|\right) = \psi_o(a)\psi_o^*(a') \prod_{n>0} \int_{-\infty}^{\infty} dx_n \chi_n(a, x_n)\chi_n^*(a', x_n)\,. \tag{2.16}$$

For tunneling from nothing ($a' = 0$) to a, we see that $\Psi(a) \simeq \rho_S(a; a' = 0)$. In the absence of the environmental degrees of freedom, Ψ reduces to the Hartle-Hawking wavefunction. Let us first consider the finite a' case. In the presence of the environment, the reduced density matrix takes the form

$$\rho_S(a; a') \simeq \psi_o(a)\psi_o^*(a') \prod_{n>0} \exp\left(-f(a, a')(a - a')^2\right). \tag{2.17}$$

Contributions to $f(a, a')$ from the tensor modes of the metric [30], massless minimally coupled inhomogeneous scalar fields [31], fermionic modes [32], Kaluza-Klein modes [57] and conformally coupled scalar fields [58] have been studied when both non-zero a and a' are in the classically allowed region. These works justify the validity of the classical time evolution of the universe. Here we use it to obtain the form of the leading decoherence term.

All these modes yield very similar results. To be specific, let us focus on the massless scalar field in the $S^4 \times M$ case, extending the result to the under-the-barrier region.

Each of its modes contributes $(a+a')^2/4a^2a'^2$ to $f(a,a')$ [31], so we have

$$\rho_S(a;a') \propto \prod_{n>0} \exp\left(-\frac{(a+a')^2(a-a')^2}{4a^2a'^2}\right) \sim \exp\left(-N\frac{(a+a')^2(a-a')^2}{4a^2a'^2}\right) \quad (2.18)$$

where N is the total number of modes included in the environment. Taking the limit of infinitely many modes would reduce the gaussian distribution to a δ-function, corresponding to an exact diagonalization of the density matrix. This means the scale factor a has been perfectly measured, which in turn implies that its momentum conjugate $-\dot{a}a$ (or simply $\dot{a}$) must have an infinite spread, which is highly non-classical (a squeezed state). For $N=0$, i.e., no modes contribute, a would have an infinite spread, a rather unrealistic, highly non-classical situation. So, on physical grounds, one expects a cut-off on the modes so that the expoenent in ρ_S (2.18) is finite. Clearly we need a regularization scheme. Fortunately, string theory provides a natural cut-off. Modes with wavelengths larger than the horizon size are not observable. Consider the above $S^4 \times M$ case. The higher multipoles can be found from an expansion of the full 3-dimensional spherical harmonics (the boundary S^3 of S^4) and normal modes in a toroidal M. So $n \to (n,l,m,n_1,n_2,\ldots,n_6)$ where (n,l,m) is for S^3 and the remaining n_i are for M. It is reasonable to only trace out modes with long wavelength λ, (say $> \bar{a}/n$ in boundary S^3, with $\bar{a}$ being some average of a and a', and $\geq L/n_i$, where $V_6 \simeq L^6$), we see that, $N \propto V_6\bar{a}^3 \simeq V_9$. Consider a field with wavenumber $k \equiv 2\pi/\lambda = 2\pi n/L$ on a torus with volume L. Since k is cut off at $k_{\max} \simeq 2\pi/l_s$, the number of modes that contributes is $N \simeq L/l_s$. Generalizing this to a higher compactified d-dimensional volume V_d, we have $N \simeq V_d/l_s^d$. For 4-dimensional de Sitter space in D-dimensional spacetime, we have

$$\mathcal{D} = cN = c\left(\frac{V_3}{l_s^3}\right)\left(\frac{V_{D-4}}{l_s^{D-4}}\right) = \frac{cV_{D-1}}{l_s^{D-1}}\,. \quad (2.19)$$

Now consider the case we are interested in, that is, the limit $a' \to 0$ in 10 dimensions. In this limit, all modes are essentially frozen. The validity of a semi-classical description of the universe requires finite spreads in both a and $\dot{a}$. To avoid either an infinitely spread in a or an infinitely spread in $\dot{a}$, we must have $N \sim V_6\bar{a}^3 \sim V_6aa'^2$. So for $a' \to 0$ and $a \simeq H^{-1} \simeq \Lambda^{-1/2}$, we have

$$\mathcal{D} \propto \frac{(V_6aa'^2)(a^2-a'^2)^2}{a^2a'^2} \to \frac{V_6}{\Lambda^{3/2}} \simeq V_9\,. \quad (2.20)$$

By considering a few universes other than $S^4 \times M$, we see that the decoherence argument agrees with the S'-brane argument, that is

$$\Psi \simeq \psi_o^*(0)\exp\left(-S_E - cV_9/l_s^9\right) \quad (2.21)$$

where $\psi_o^*(0)$ provides the normalization.

Note that the actual determination of $\mathcal{D}$ requires knowing the full wavefunction $\Psi(a=0)$ (2.15): this is the wavefunction when there is no classical spacetime. Since classical spacetime is fundamental in general relativity, $\Psi(a=0)$ may not be well-defined in quantum gravity. It will be interesting to see if string theory can fix $\Psi(a=0)$, or it has to be postulated. The parameter c actually depends on the particular vacuum in string theory. It is presumably a good approximation to take c to be constant for a subclass of vacua.

The inclusion of $\mathcal{D}$ should suppress instantons with more complicated geometries, such as those considered in Ref [28]. Higher order terms involving higher powers of the Riemann tensor and field strengths are present in low-energy effective theory in string theory, so they should be included in S_E [59, 60]. In the rest of this paper, we shall ignore such terms and the higher order terms in $\mathcal{D}$ as well as the variation of c. This is certainly sufficient for illustrative purposes.

3. Toy models: ten dimensional gravitational instantons

To begin, we study a few instanton solutions in ten dimensions. We will be interested in S^{10} as well as $S^4 \times M$ solutions in which four (1 time and 3 space after Wick rotation) out of the ten dimensions inflate while M of the remaining six spatial dimensions have static behavior during an early inflationary phase. These different solutions will have different topologies for the extra six dimensions.

3.1 S^{10}

This instanton solution is a trivial generalization of the S^4 instanton. The euclidean metric ansatz for S^{1+n} is:

$$ds^2 = dt^2 + a(t)^2\left(\frac{dr^2}{1-r^2} + r^2 d\Omega^2_{(n-1)}\right). \tag{3.1}$$

The euclidean Einstein equations for S^{1+n}, for any n, are:

$$\begin{aligned} -\frac{1}{2}n(n-1)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) &= \Lambda \\ -(n-1)\frac{\ddot{a}}{a} - \frac{1}{2}(n-1)(n-2)\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) &= \Lambda\,. \end{aligned} \tag{3.2}$$

The bounce solution is given by $a(t) = H^{-1}\cos(Ht)$ with $H^2 = 2\Lambda/n(n-1)$ and $R = n(n+1)H^2 = 2(n+1)\Lambda/(n-1)$. For the case of S^{10}, this becomes $H^2 = \Lambda/36$. By Wick rotating the time axis one gets a closed 9 spatial dimensional inflating universe with the scale factor given by $a(t) = H^{-1}\cosh(Ht)$.

Now, in string theory, Λ can come from the presence of extra branes. Starting with a supersymmetric vacuum, we can add N pairs of $D9-\bar{D}9$ branes, where the vacuum energy ρ_{vacuum} is just $2N$ times the $D9$-brane tension,

$$\Lambda = N\Lambda_1 = 8\pi G_{10}\rho_{\text{vacuum}} = 16\pi G_{10} N\frac{M_s^{10}}{(2\pi)^9 g_s} \tag{3.3}$$

where g_s and $M_s = 1\sqrt{/\alpha'}$ are the string coupling constant and the string mass scale, respectively. For numerical calculations, we shall take $g_s = 1$. The ten dimensional Newton's constant is given by

$$8\pi G_{10} = \frac{g_s^2 (2\pi)^7}{2M_s^8} . \tag{3.4}$$

The entropy of a de Sitter space equals the euclidean action, $S_{entropy} = -S_E$. In the presence of matter fields, matter excitation modes can be present. However, it is known that the pure de Sitter space saturates the entropy bound [61], so we expect the pure de Sitter instanton to have the largest tunneling probability.

In string theory, the dilaton plays a crucial role in the gravity sector. A summary of the analysis including the dilaton can be found in appendix D. The inclusion of the dilaton field does not change the qualitative features we are interested in.

3.2 $S^4 \times S^6$, $S^n \times M$ etc.

We start with the 10-dimensional euclidean metric ansatz for $S^{1+n_1} \times S^{n_2}$:

$$ds^2 = d\tau^2 + a(\tau)^2 \left(\frac{dr^2}{1-r^2} + r^2 d\Omega^2_{(n_1-1)} \right) + b(\tau)^2 \left(\frac{d\rho^2}{1-\rho^2} + \rho^2 d\Omega^2_{(n_2-1)} \right) . \tag{3.5}$$

The solution of the corresponding Einstein equation requires b to be constant (see appendix E) with

$$\begin{aligned} b^2 &= \frac{(n_2-1)(n_1+n_2-1)}{2\Lambda} \\ H^2 &= \frac{2\Lambda}{n_1(n_1+n_2-1)} . \end{aligned} \tag{3.6}$$

For $n_1 = 3$, this yields a 4-dimensional de Sitter space and a static S^6. That is, if we consider time to lie in S^4 after Wick rotation, then it describes a ten dimensional universe with inflation occuring in the $3+1$ dimensions corresponding to S^4 while the remaining 6 spatial dimensions in S^6 remain static. If we include a small time varying component to $b(\tau)$, it will be rapidly inflated away.

It is easy to generalize the above analysis to other cases. In general, for $S^{n_1} \times S^{n_2} \times S^{n_3} \times \cdots$, when rotated to lorentzian time, only the spatial directions in S^{n_i} which contains the time direction will inflate, while all the rest of the spatial directions remain static.

We tabulate the probabilities thus calculated for the different geometries. (The list is incomplete and is representative only.)

For very small $\hat{c}$, $N_m = 1$ is the optimal choice (since N_m is an integer); that is, one brane pair maximizes $\mathcal{F}$. In this case, we have $\mathcal{F} \simeq -S_E \sim 10^9$, independent of the value of $\hat{c}$. Let us make some comments here:

1. In string theory, there are tunnelings to many possible expanding universes. So the relative tunneling probabilities become an important issue to pick out the most likely universes.

2. There are at least 2 possible ways to stabilize the moduli in superstring theory: dynamical versus cosmological. Cosmological stabilization has been explored in the presence of string winding modes [62] and more recently in the presence of branes [63]–[66]. Dynamical stabilization has made substantial progress recently [2, 3]. In this scenario, the universe is sitting at a metastable vacuum, with a tiny positive cosmological constant. As pointed out earlier, there is a true (supersymmetric) vacuum with zero cosmological constant where all the extra dimensions are decompactified. Even if we believe in dynamical stabilization, the universe must avoid an early phase where the extra dimensions go through a rapid growth (as illustrated in figure 4). So cosmological stabilization as presented above may be necessary even in the presence of eventual dynamical stabilization.

3. The simplest instanton solution is the de Sitter S^{10} instanton. Such a solution represents the tunneling of the universe to a space-filling $D9-\bar{D}9$ system in which all the 9 spatial dimensions inflate as the brane and the antibrane decay down the tachyon potential. The inflation offered by the $D9-\bar{D}9$ system is, however, very marginal, amounting only to a fraction of an e-fold. As this system decays into pairs of $D7-\bar{D}7$ and radiation, the 9 spacial dimensions undergo a radiation dominated expansion and this leads to a significant separation between the $D7-\bar{D}7$ pairs. This sizeable separation may lead to a significant number of e-folds in the ensuing inflation between the $D7-\bar{D}7$ system.

4. If we use Linde/Vilenkin's wavefunction (see appendix A for a brief review), $\psi \sim e^{-|S_E|} \sim e^{-10^9/N}$, so it seems that $(S^2)^5$ among the list in table 1 is preferred in this case. However, an inflationary universe with large N (i.e., large cosmological constant) is also preferred. Of course, for large N, the semi-classical approximation breaks down and we lose all control of the estimate. Furthermore, the inclusion of the decoherence term would not help.

Other interesting instanton solutions involve geometries of the form $S^m \times T^1 S^n \times T^k$ are discussed in appendix E.

3.3 An application of the new wavefunction

Consider the function $\mathcal{F}$. If the cosmological constant is provided by N $D9$ brane-antibrane pairs, $\mathcal{F}$ can be written as:

$$\mathcal{F} = \frac{1}{16\pi G_{10}} \frac{N\Lambda_1}{2} V_{10} - \hat{c} M_s^9 V_9 \,. \tag{3.7}$$

Considering geometries of the form $S^{1+n_1} \times S^{n_2}$:

$$\mathcal{F} = \frac{N\Lambda_1}{32\pi G_{10}} V_{1+n_1} V_{n_2} - \hat{c} M_s^9 V_{n_1} V_{n_2} \tag{3.8}$$

where V_{1+n_1} and V_{n_2} are the surface areas of S^{1+n_1} and S^{n_2}, respectively. V_{n_1} is the area of the equator of S^{1+n_1}. One should note that the surface areas depend on N via the Hubble

Geometry	$-S_E \times 10^{-9}$	Volume $\times 10^{-15}$	$\frac{N_m}{\hat{c}^2} \times 10^{-12}$	$\mathcal{F}\hat{c}^8 \times 10^{40}$
S^{10}	3.94170	3.92236	1.25626	1.75627
$S^8 \times S^2$	2.88416	2.89347	1.27620	1.20694
$S^7 \times S^3$	3.07644	3.11670	1.29131	1.23300
$S^6 \times S^4$	3.17258	3.20829	1.31267	1.17892
$S^5 \times S^5$	3.17258	3.26494	1.34511	1.07489
$S^5 \times S^3 \times S^2$	2.24004	2.30866	1.34511	0.760062
$S^4 \times S^6$	3.17258	3.31351	1.40018	0.910688
$S^4 \times S^3 \times S^3$	2.40348	2.49832	1.40018	0.686643
$S^4 \times S^2 \times S^2 \times S^2$	1.50938	1.59048	1.40018	1.07489
$S^3 \times S^2 \times S^2 \times S^3$	1.58630	1.73150	1.51325	0.335524
$S^2 \times S^6 \times S^2$	2.11506	2.55074	1.86691	0.192098
$S^2 \times S^4 \times S^4$	2.26888	2.75480	1.86691	0.207466
$S^2 \times S^2 \times S^2 \times S^2 \times S^2$	1.00946	1.22435	1.86691	0.092207

Table 1: The time coordinate is part of the first factor S^n. The euclidean action S_E is for a single pair of $D9 - \bar{D}9$ branes. The 9-dimensional volume of the boundary is measured in units of string scale, that is Volume= $M_s^9 V_9$. The value of N_m is for maximizing $\mathcal{F}$, the value of which is given in the last column. For small $c \equiv (2\pi)^9 \hat{c}$, integer $N_m = 1$ maximizes $\mathcal{F}$, which has its value very close to $-S_E$, not that given in the last column.

radius, H^{-1}. To find the value of $N = N_m$ at which $\mathcal{F}$ is a maximum, we just require that $\partial \mathcal{F}/\partial N = 0$. This gives the following value for N:

$$\frac{N_m}{\hat{c}^2} = \frac{81}{64}(2\pi)^{16} \frac{\nu(n_1)^2}{n_1 \nu(1+n_1)^2} \tag{3.9}$$

where $\nu(n)$ is the "surface area" of a unit n-sphere:

$$\nu(n) = \frac{2\pi^{(1+n)/2}}{\Gamma((1+n)/2)} \,. \tag{3.10}$$

One can plug this value of N back in the expression for $\mathcal{F}$ and get the extremized value of $\mathcal{F}$:

$$\mathcal{F}_{\max} = \frac{2^{33}}{3^{18}(2\pi)^{63}} \left(\frac{n_2 - 1}{n_1}\right)^{n_2/2} n_1^9 \frac{\nu(n_2)[\nu(1+n_1)]^9}{[\nu(n_1)]^8} \frac{1}{\hat{c}^8} \,. \tag{3.11}$$

Though all geometries of type $S^{1+n_1} \times S^{n_2}$ have the same dependence of $\hat{c}$ (viz. $1/\hat{c}^8$), they have different coefficients in front of the $\hat{c}^{-8}$ term. In fact, the volume term ($\hat{c} M_s^9 V_9$) lifts the degeneracy between the $S^m \times S^n$ and the $S^n \times S^m$ geometries.

We see in table 1 that, irrespective of the value of $\hat{c}$, tunneling via S^{10} has the largest probability. This is not surprising, since S^{10} is the most symmetric instanton. S^{10} tunneling implies that the universe begins with all its 9 spatial dimensions uncompactified. Including the dilaton will not change this qualitative feature. Fortunately, as we shall see that, in string theory, the tunneling to a realistic inflationary scenario with 6 dimensions compactified is preferred over S^{10}.

A complete understanding of quantum gravity is lacking. However, as long as the quantum corrections are small, one can justify the semiclassical treatment that we have employed. The curvature of the instanton solutions is given by $R = 5\Lambda/2 = (5N/8\pi^2)g_s M_s^2$. The semiclassical description is reliable as long as $R/M_s^2 \simeq 5Ng_s/8\pi^2 \ll 1$, or $Ng_s \ll 15$. This implies that, for $g_s \simeq 1$, $N \sim 1$. In table 1, we see that

$$N_m \simeq \hat{c}^2 \times 10^{12} \,. \tag{3.12}$$

For the above semiclassical analysis to be meaningful, this condition requires $\hat{c} \lesssim 10^{-6}$, or

$$c \equiv (2\pi)^9 \hat{c} \lesssim 10 \tag{3.13}$$

which we shall assume is satisfied. For $c \ll 10$, we have $N_m = 1$ (since N is quantized and $N = 0$ is ruled out). In this case, the value of $\mathcal{F}$ is actually very close to $-S_E$, since the $\mathcal{D}$ term is completely negligible. As we shall see, the phenomenological analysis below puts $c \simeq 10^{-3}$. In any case, the $S^4 \times M$ instanton like those in table 1 is never preferred.

4. Tunneling to inflationary universes in string theory

Now we turn to more realistic vacua in string theory. First, we consider tunneling to string vacua that initiate an inflationary phase. We then consider tunneling directly to string vacua that mimic our universe today (i.e., with a very small cosmological constant). To be specific, we shall study the model due to Giddings etc. [1] and Kachru etc. [2, 22]. It will be important to study as many stringy vacua as possible. However, the analysis here should be sufficient to illustrate our approach.

Start with a 4-fold Calabi-Yau manifold in F-theory, or, equivalently, a type-IIB orientifold compactified on a 3-fold Calabi-Yau manifold with fluxes to stabilize all but the volume modulus [1]. For large volume, supergravity provides a good description. The presence of $D7$-branes introduces a non-perturbative superpotential W that stabilizes the volume modulus in a supersymmetric AdS vacuum. The introduction of $\bar{D}3$-branes in a warped type-IIB background breaks supersymmetry and lifts the AdS vacuum to a metastable de Sitter (dS) vacuum [2] (the KKLT vacuum). To realize inflation, KKLMMT [22] introduces a $D3$-$\bar{D}3$-brane pair, whose vacuum energy drives inflation [67]–[70]. The $\bar{D}3$-brane is fixed with the other $\bar{D}3$-branes in the Klebanov-Strassler deformed conifold [71] and the inflaton ϕ is the position (relative to the $\bar{D}3$-branes) of the $D3$-brane. This yields a potential for the mobile $D3$-brane in this $D3$-$\bar{D}3$-brane inflationary scenario

$$V(\rho, \phi) = V_F(\mathcal{K}(r), W(\rho, \phi)) + \frac{D}{r^2} + V_{D\bar{D}}(\phi) \tag{4.1}$$

where r is the physical size of the compactified volume and ρ the corresponding bulk modulus. $V_F(\mathcal{K}(r), W(\rho, \phi))$ is the F-term potential, where the superpotential W is expected to stabilize the volume modulus in an AdS supersymmetric vacuum, while the $\bar{D}3$-branes (the Dr^{-2} term) in the warped geometry breaks supersymmetry to lift the AdS vacuum to a de Sitter vacuum (a metastable vacuum with a very small cosmological constant and a lifetime larger than the age of the universe). Note that this non-perturbative interaction term leaves ϕ to be essentially massless, that is, there exists a shift symmetry [23]. This shift symmetry is broken by the $D3$-$\bar{D}3$ potential $V_{D\bar{D}}(\phi)$, which is very weak due to warped geometry. This inflaton potential is designed to break the shift symmetry slightly, so inflation can end after slow-roll.

We shall study the wavefunction of tunneling to this type of string states. We shall find the maximum $\mathcal{F}$ for such inflationary vacuum as well as for the corresponding vacua without the extra pair of $D3$-$\bar{D}3$-brane needed for inflation. In each case, the fluxes are determined by maximizing $\mathcal{F}$. If we know $\hat{c}$, this ansatz will select a specific stringy vacuum via quantum tunneling. Since we do not know the value of $\hat{c}$, we shall instead use the data to fix it. Having fixed $\hat{c}$, we can then evaluate $\mathcal{F}$ for any vacuum. Fortunately, the qualitative features are unchanged if we vary $\hat{c}$ by a few orders of magnitude. Before going into the discussions, we give a brief summary of the key results.

- Using G_N and the COBE density perturbation data, M_s and c can be fixed. We find $c \sim 10^{-3}$. Maximizing $\mathcal{F}$ allows us to fix the values of the fluxes.

- We find that tunneling to an KKLMMT-like inflationary universe (with $\mathcal{F} \sim 10^{18}$) is much preferred over the S^{10} universe (with $\mathcal{F} \sim 10^{9}$) discussed earlier.

- This $D3$-$\bar{D}3$ inflationary universe is also much preferred over the KKLT vacua without inflation ($\mathcal{F} < 0$). That is, direct tunneling to today's vacuum is strongly suppressed.

- The KKLT string vacuum state that the inflationary universe ends in (after the $D3$-$\bar{D}3$ branes have annihilated) is different from the vacuum that will be reached directly from tunneling. They have different fluxes.

- More detailed analysis of the above model and investigation of other string vacua should help to narrow the uncertainty in c. A better understanding of Ψ will allow us to perform a more precise selection of a stringy vacuum state.

4.1 Set-up

The moduli stabilization follows the approach due to GKP [1] and KKLT [2], where more background and details can be found. Here we give an overall picture of the application to the wavefunction. Many details are saved for appendix F. The complex structure moduli are stabilized by the introduction of quantized fluxes wrapping around some cycles while the Kähler modulus is stabilized by a non-perturbative QCD-like effect. In particular, let M be the units of RR 3-form field strength in a 3-cycle A and $-K$ be the units of NS-NS 3-form field strength in the dual 3-cycle B. Here, K and M are discrete parameters. Different choices of K and M correspond to different vacua. For fixed K and M, the introduction of an additional pair of $D3-\bar{D}3$-branes [22] raises the vacuum energy and inflation takes place. It is argued [23] that the slow-roll condition is generically satisfied so there will be enough inflation before the pair of branes collides, annihilates, (re)heats and starts the hot big bang epoch.

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Start with the metric

$$ds_{10}^2 = e^{2u(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2u(y)}\hat{g}_{mn}dy^m dy^n \tag{4.2}$$

where $A(y)$ measures the initial size of the universe created via tunneling. Consider the simple case of a single complex modulus z. There are $2 + 2b_{2,1} = 4$ 3-cycles, namely a pair (A, B) (dual cycles that intersect only once) and an additional pair (A', B'). The integrals are the periods defining the complex structure of the conifold. In particular, z is the complex coordinate for the cycle A

$$z \equiv \int_A \Omega \tag{4.3}$$

where Ω is the $(3,0)$-form and that on the dual cycle is

$$\mathcal{G}(z) \equiv \int_B \Omega = \frac{z}{2\pi i}\ln z + \text{holomorphic}\,. \tag{4.4}$$

The three-cycles A and B are in the vicinity of the Klebanov-Strassler conical point where fluxes are turned on:

$$\begin{aligned} \frac{1}{2\pi\alpha'}\int_A F_{(3)} &= 2\pi M \\ \frac{1}{2\pi\alpha'}\int_B H_{(3)} &= -2\pi K\,. \end{aligned} \tag{4.5}$$

where $F_{(3)}$ is the 3-form field strength of the RR field and $H_{(3)}$ is the 3-form field strength of the NS-NS field. Turning on $-K'$ units of $H_{(3)}$ on the B' cycle, we finally obtain the superpotential

$$W = M\mathcal{G}(z) - K\tau z - K'\tau z'(z) + Ae^{ia\rho} \tag{4.6}$$

where $\tau = C_{(0)} + ie^{-\phi}$ and the last term is a non-perturbative interaction contribution due to the presence of D7-branes. Here, z' is a function of z which is generically non-vanishing at $z = 0$, $z'(0) \simeq 1$.

Following KKLMMT we have

$$\Lambda = 8\pi G\, T_3 \frac{r_0^4}{R^4}\left(1 - \frac{1}{N}\frac{r_0^4}{r_1^4}\right) \tag{4.7}$$

where r_0 and r_1 are the positions of the stack of $\bar{D}3$-brane and the $D3$-brane, respectively. T_3 is the D3-brane tension:

$$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2} \tag{4.8}$$

and R is the curvature radius of the AdS geometry:

$$R^4 = 4\pi g_s N\alpha'^2\,. \tag{4.9}$$

Here N is the number of $\bar{D}3$-branes. The $D3$ charge conservation requires

$$N = \frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{M}} H_{(3)} \wedge F_{(3)} = MK\,. \tag{4.10}$$

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To trust the low energy supergravity approximation, we assume $N \gg 1$. After compactification of the internal space we have

$$M_{Pl}^2 = \frac{2}{(2\pi)^7} \frac{V_6}{g_s^2 \alpha'^4} \tag{4.11}$$

where $M_{Pl}^{-2} \equiv 8\pi G_N$ and V_6 is the volume of the Calabi-Yau manifold. For simplicity we consider $V_6 = \alpha'^3 r^6$, where r is related to the imaginary part of the Kähler modulus $\rho = b + ir^4$.

Considering first only the imaginary part of ρ, we find

$$-S_E = \frac{3\pi}{2G_N \Lambda} = \frac{6}{(2\pi)^9} \left(\frac{\rho}{g_s}\right)^3 \left(\frac{R}{r_0}\right)^4 . \tag{4.12}$$

Fluxes induce a large hierarchy of scale between r_0 and R: $(\frac{r_0}{R})^4 = e^{-8\pi g_s K/3M}$. We see that S_E (4.12) is exponentially large, due to the warp factor $\frac{r_0}{R}$. For example, adopting the numbers in appendix C of KKLMMT, we find that $-S_E \sim 10^{21}$. This implies that, order-of-magnitude wise, $\mathcal{F} \lesssim 10^{21}$.

4.2 Applying the new wavefunction

Now we are ready to apply our ansatz to the tunneling from nothing to the above inflationary vacuum in string theory. Since the extra dimensions are stabilized already, we may assume that the terms needed to suppress large compactification sizes are negligible. Keeping the leading term in $\mathcal{D}$, we have $\mathcal{F} \sim e^{-S_E - \hat{c} M_s^9 V_9}$, where V_9 is the spatial volume of the closed universe at the end of tunneling.

For $S^4 \times CY$, as long as the CY volume modulus and complex structures are fixed, we can use the effective four dimensional point of view such that $V_9 = V_6 \times V_3$, where V_6 and V_3 are the spatial volumes of the CY manifold and our three dimensional space, respectively. We have

$$V_3 = \frac{2\pi}{3} \left(\frac{\Lambda}{3}\right)^{-\frac{3}{2}} . \tag{4.13}$$

Using eqs. (4.7), (4.8) and (4.11) with the identity $V_6 \equiv \alpha'^3 r^6 = \alpha'^3 \rho^{\frac{3}{2}}$, we find

$$\mathcal{D} = \hat{c} M_s^9 V_9 = \hat{c} \frac{2\sqrt{3}}{(2\pi)^5} \rho^{\frac{15}{4}} g_s^{\frac{-3}{2}} \left(\frac{R}{r_0}\right)^6 . \tag{4.14}$$

Maximizing Ψ or $\mathcal{F}$ determines M to be M_1 and K to be K_1. The details of the analysis can be found in appendix F. For any given vacuum state, quantities like $z'(0)$, $|\mathcal{G}(0)|$, a and A etc. are all in principle calculable. For our purpose here, we shall simply adopt some generic choices of values.

In GKP it was assumed $|\mathcal{G}(0)| \sim 1$ and $z'(0) \sim 1$, which we shall adopt. In this approximation, we find, in appendix F,

$$\begin{aligned}
M_1 &\sim 0.4 \times |A| \\
K_1 &\sim 0.8 \times \ln\left(10^{-4} a^{3/4}\, |A|^{3/2}\, \hat{c}^{-1}\right) \\
\left(\frac{r_0}{R}\right) &\sim 115 \times a^{-3/8}\, |A|^{-3/4}\, \hat{c}^{1/2} \\
\frac{M_{Pl}}{M_s} &\sim 3 \times 10^{-4}\, a^{-3/4}\, |A| \\
\frac{1}{g_s} &\sim 0.1 \times |A| \\
r &\sim \left(\frac{1.2}{a}\right)^{1/4} .
\end{aligned} \tag{4.15}$$

The value of $\mathcal{F}$ at the maximum is

$$\mathcal{F}_{\max} \sim 2 \times 10^{-18}\, a^{-3/2}\, |A|^6\, \hat{c}^{-2} . \tag{4.16}$$

or, expressing in terms of the COBE normalization, δ_H:

$$\mathcal{F}_{\max} \sim 1.6 \times 10^4\, \delta_H^{-3} . \tag{4.17}$$

For $\delta_H \sim 2 \times 10^{-5}$, we find

$$\mathcal{F}_{\max} \sim 10^{18} . \tag{4.18}$$

To trust the low energy supergravity approximation, we need $r \gg 1$ and $g_s < 1$. The first condition requires $a \ll 1$. For example the value chosen by KKLT, $a = 0.1$, results in $r \sim 2$. One may choose even smaller value of a to improve the low energy supergravity approximation. To satisfy the condition $g_s < 1$, we simply choose $|A| > 10$.

We see that all the physical outputs are functions of a, $|A|$ and $\hat{c}$. As an example, let us take $a = 0.001$, $|A| = 100$. To get the COBE bound $\delta_H \sim 10^{-5}$, we find

$$\hat{c} \sim 10^{-10} \quad \rightarrow \quad c \sim 10^{-3} .$$

With above values of a and $|A|$, we get $M_1 \sim 40$ and $K_1 \sim 12$, $\frac{r_0}{R} \sim 2 \times 10^{-3}$ and $M_{Pl} \sim 6\, M_s$. The closeness of M_s to M_{Pl} may raise some concerns initially, but to find the actual physical scale of inflation one must take into account the warping effect. Including the effect of the warp factor, one finds

$$\rho_v^{\frac{1}{4}} \sim \left(\frac{r_0}{R}\right) M_s \sim 10^{14}\,\mathrm{GeV} \tag{4.19}$$

in agreement with the commonly used energy scale of inflation. Summarizing the main results, we find that

1. Using only the Hartle-Hawking wavefunction, the flux M is fixed to a finite value (of order 10) while the flux K is not determined. The larger is the value of K, the larger is the tunneling probability. Since large K exponentially suppresses the vacuum energy, we end up with an universe with a very small cosmological constant, as expected.

2. Using our improved wavefunction, maximizing the tunneling probability to an inflationary vacuum determines both $M = M_1$ and $K = K_1$. For a reasonable choice of $c \sim 10^{-3}$, the preferred inflationary universe (that is, with the largest tunneling probability) easily fits the COBE data, with $\mathcal{F} \simeq 10^{18}$.

3. Using the above value for c, the 10-dimensional de Sitter universe and similar vacua have $\mathcal{F} \simeq 10^9$.

4. The KKLT vacua are very much like our today's universe. It is easy to estimate the tunneling probability to such a vacuum state, as done in appendix F. We find that $\mathcal{F}_{KKLT} < 0$, so they have extremely small tunneling probabilities. Taking Λ to be today's dark energy value, we have $\mathcal{F} \sim -10^{170}$.

5. To conclude, the universe prefers to go through an inflationary phase. This qualitative (order of magnitude) result is largely independent of the details (see figure 3). In this sense, we believe the picture is robust.

6. Let K_0 and M_0 be the values for the vacuum with the largest tunneling probability to go directly to an universe bypassing an inflationary phase. We find that $(M_1, K_1) \neq (M_0, K_0)$. This means that vacua (such as the (M_0, K_0) vacuum) without an inflationary road leading to them may be unreachable.

7. The above analysis is very crude. Note that A and a are input parameters not determined by maximizing $\mathcal{F}$. Also, instead of order of magnitude estimates, functions like z', $\mathcal{G}$ etc. should be calculable. With a better wavefunction Ψ, we should be able to find the preferred inflationary universe with no free parameters. It will be interesting to find $\mathcal{F}$ for other inflationary scenarios in string theory [4, 25].

8. Although we find $c \sim 10^{-3}$, the analysis is quite crude. As is clear in the analysis, some reasonable variation of the parameters used can easily change c by a few orders of magnitude. We believe $c \sim 1$ is quite possible. This calls for a more careful study.

5. Supercritical versus critical string vacua

Although string theory is known to have 10 (11 for M theory) as its critical dimension, higher-dimensional vacua are known to be possible [47, 48, 49]. In $D > 10$ dimensions, the low energy effective theory containing the massless graviton, dilaton and R-R fields is given by (in Einstein frame) [47]

$$S_D = \frac{1}{2{\kappa_D}^2} \int d^D x \sqrt{-g} \Big(R - \frac{2(D-10)}{3\alpha'} e^{4\phi/(D-2)} - \\ - \frac{4}{D-2} \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{2} \sum_p e^{4(1-p)\phi/D-2} (F_p)^2 \Big) \tag{5.1}$$

where the sum runs over various RR fields in the theory, ϕ is the dilaton and κ_D stands for D-dimensional gravitational constant:

$$2\kappa_D^2 \equiv 16\pi G_D = g_s^2 (2\pi)^{D-3} M_s^{2-D} \equiv \frac{g_s^2 l_s^{D-2}}{2\pi} \,. \tag{5.2}$$

Note the presence of the cosmological constant term, due to the linear dilaton background. This term is absent in $D = 10$, allowing $D = 10$ flat spacetime. For $D > 10$, 4-dimensional flat, or almost flat, spacetime may be achieved in the presence of contributions from the many RR fluxes, branes and orientifold planes. In general there are many local dS vacua along with a global minimum, corresponding to $\phi \to -\infty$. The resulting cancellation can in principle yield an almost stable vacuum with a parametrically small cosmological constant that attains the value of the observed dark energy [48, 49].

Such supercritical vacua in general do not have supersymmetry. So the liklihood of low energy supersymmetry, a very important phenomenological issue, hinges crucially on which type of vacua nature prefers, critical or supercritical. We like to check if SOUP shed light on this fundamental question. A general study in SOUP program to find the dimensionality of the space-time is rather delicate and it may need a good knowledge of detailed properties of each vacuum. However, as a first test to illustrate the proposal, let us consider the D-dimensional instanton S^D for $D > 10$. The main contribution for the cosmological constant is given above

$$\Lambda(D) = \frac{2(D-10)}{3\alpha'} g_s^{\frac{4}{D-2}} . \tag{5.3}$$

We would like to calculate $\mathcal{F}$ for different S^D for $D > 10$, and compare it to the value of $\mathcal{F}$ for S^{10}, the critical case.

Using eq. (3.10) for the surface area of a unit S^D instanton along with eqs. (5.2), (5.3), we find

$$\begin{aligned}\mathcal{F} = {} & 4\pi^2\sqrt{\pi} g_s^{-2} \left(\frac{3}{2(D-10)}\right)^{\frac{D-2}{2}} \left(\frac{(D-1)(D-2)}{8\pi}\right)^{\frac{D-1}{2}} \times \\ & \times \left[4\sqrt{\frac{\pi(D-1)}{2(D-2)}}\frac{1}{\Gamma(\frac{D+1}{2})} - \frac{c}{(2\pi)^2 g_s^{(D-2)/2}}\sqrt{\frac{3}{2(D-10)}}\frac{1}{\Gamma(\frac{D}{2})}\right] .\end{aligned} \tag{5.4}$$

Using $c \sim 10^{-3}$ and $g_s = 1$ we find that $\mathcal{F}$ quickly goes to zero for large D: ($\mathcal{F} \simeq 10^9$ for $D = 10$, $\mathcal{F} \simeq 10^4$ for $D = 11$, $\mathcal{F} \simeq 10^3$ for $D = 12$, $\mathcal{F} \simeq 12$ for $D = 16$ and $\mathcal{F} \simeq 0.7$ for $D = 20$.) This is consistent with the general expectation that for large Λ, $\mathcal{F} \to 0$, as is indicated in figure 3. Adding a non-BPS brane or brane pair will simply decrease $\mathcal{F}$ further. As we increase D, the semi-classical approximation eventually breaks down, but we do not expect any change in the qualitative picture. Interestingly enough, we see that $\mathcal{F}$ for S^D, $D > 10$ is negligible compared to the value $\mathcal{F} \sim 10^9$ for S^{10}, or to the KKLMMT inflationary model with $\mathcal{F} \sim 10^{18}$. Naively, we may conclude that critical string vacua are preferred over supercritical string vacua. Unfortunately, such a conclusion is premature.

It remains to be seen if some sort of brane inflationary scenario may be implemented in the supercritical string framework. This is a big if. Suppose a brane inflationary scenario similar to that discussed earlier can be implemented. Let us further assume that both inflationary scenarios (in critical and in supercritical strings) that agree with our early universe are among the vacua preferred by SOUP. Can SOUP then distinguish between them ? In this case, it turns out that higher correction terms to $\mathcal{F}$ is required to distinguish them. To see why, let us start with $G_N = G_D/V_{D-4}$, where V_{D-4} is the volume of the compactified dimensions. Using eq. (4.13), a simple generalization of the earlier discussions gives

$$\mathcal{F} = \frac{3\pi V_{D-4}}{2G_D\Lambda} - c\frac{2\pi V_{D-4}}{3(\Lambda/3)^{3/2} l_s^{D-1}} = \frac{3\pi}{2G_N\Lambda} - c\frac{g_s^2}{48\pi G_N l_s (\Lambda/3)^{3/2}} \,. \tag{5.5}$$

For D-independent c, as suggested by the decoherence argument, there is no explicit D-dependence in $\mathcal{F}$. With $c \sim 10^{-3}$, maximizing $\mathcal{F}$ yields $\sqrt{\Lambda} \sim 10^{14}\,\mathrm{GeV}$ and $\mathcal{F} \sim 10^{18}$, independent of D. To distinguish them, we may need a better knowledge of $\mathcal{F}$. It is also important to check if a viable inflationary scenario may be incorporated into supercritical string vacua. In that case, maximizing $\mathcal{F}$ and fitting data will determine g_s and Λ. We can then compare the probabilities of tunneling to such a supercritical inflationary scenario and to the most favorable critical inflationary scenario.

For example, fixing l_s, c and setting $\partial\mathcal{F}/\partial\Lambda = 0$ yields $\mathcal{F} \propto g_s^{-4}$. With explicit supercritical inflationary scenarios, SOUP may be able to express its preference between critical and supercritical inflationary vacua. If, on the other hand, it is $\hat{c}$ instead of c that is D-independent, then $\mathcal{F} \propto g_s^{-4}(2\pi)^{-2(D-10)}$. In this case, critical string vacua are likely to be preferred. It is clear that further study will be important to find the preferred vacuum.

6. Remarks

In this paper, we propose an improvement of the Hartle-Hawking wavefunction. The key difference in the new wavefunction is the inclusion of the the backreaction effects due to the interaction of the metric with the matter fields. These matter fields (multipole modes perturbing the background classical metric as well as other bosonic and fermionic modes) play the role of the environment and are traced over, providing a decoherence term that tends to suppress the tunneling to universes with small cosmological constants. As a consequence, intermediate values of the vacuum energy seems to be preferred. This is precisely what we want, since this allows the tunneling to an inflationary universe not unlike the one our universe has gone through. To conclude, let us make some remarks:

- Here we like to point out that the above proposal should be further modified. Consider an instanton that leads to a universe with little or no inflation. That universe will either recollapse, or reach only a very small size. As a result, the likelihood for us to be inside such a region will be extremely small compared to a universe that goes through an extended period of inflation. As a simple ansatz, the likelihood P_S of an observer ending in a particular universe S with size V_S should be

$$P_S = \frac{V_S}{V_I}|\mathcal{P}|^2 e^{2\mathcal{F}} \tag{6.1}$$

  where V_I is the volume of the initial universe just after tunneling, and V_S is the volume immediately after inflation. Here $\mathcal{P}$ is the prefactor that is calculable in principle.

- Following the Hartle-Hawking prescription, it seems that a universe with an arbitrarily small cosmological constant is preferred. This corresponds to an arbitrarily large instanton, which corresponds to infinite (euclidean) time for the "nucleation" bubble creation. This is clearly unacceptable, because it implies that the universe as big as today's universe is much more likely to be created directly from nothing than a much smaller universe, say our universe 13 billions years ago. We argue that there must exist a mechanism that suppresses this effect, such that a vacuum energy density acceptable for inflation is naturally selected. Including the coupling of matter fields and higher gravitational modes, we are led to a modified wavefunction, which includes a decoherence term. It is important to give a more complete argument for the wavefunction we proposed, and if additional modifications should be included. It will be a challenge to calculate the value of the coefficient c.

- String theory has many vacua/state with very different vacuum energies. We use the proposed wavefunction to compare the probability of tunneling from nothing to various geometries, albeit a limited set. By comparing the tunneling probabilities for the different geometries, we see that tunneling from nothing to an inflationary universe very close to what our universe went through is favored. That is, tunneling directly to a universe with a much smaller cosmological constant (like today's dark energy) is very much suppressed. Tunneling directly to a 10-dimensional de Sitter universe is also suppressed.

- Not all viable string vacua allow an extended period of inflation that ends with a hot big bang. Only those string vacua that are sitting at the end of a long inflationary road have any chance of being our vacuum. That is, we will not end up in a vacuum which is not close to a favorable inflationary road. In the search of why our particular vacuum is selected, this should cut down substantially the number of string vacua one has to take into consideration.

- So far, we have considered only a very limited set of geometries. It will be important to check if a universe with 3 inflating dimensions is really preferred over all other geometries. For example, tunneling to a Randall-Sundrum brane world (i.e., a Horava-Witten model) [72, 73] is worth a new analysis within SOUP. Clearly more investigation is required with realistic string models to see if our particular 4-dimensional universe is favored over other 4-dimensional universes.

- For SOUP to be phenomenologically meaningful, we must use the Hartle-Hawking wavefunction and not the alternative proposed in ref. [33, 34]. In practical terms, one may view this as a resolution of the 20 year old debate.

- Since it is clear that the string theory has many instanton solutions, we must entertain possibility of a collection of instantons each of which gives rise to its respective universe. This also suggests that, semiclassically the wavefunction should include the sum over the metric that includes disconnected pieces. This must allow the collision of different universes. In terms of string theory, presumably Ψ is a wavefunction in string field theory. Semiclassically, one may treat closed string modes, in particular gravity, as background fields with small fluctuations. Such fluctuations allow topology changes and so the interaction among the topologically distinct universes should be included. This is clearly interesting to investigate further.

- $S_{entropy} = -S_E$ is the entropy of de Sitter space, maximizing the tunneling probability with the Hartle-Hawking wavefunction is equivalent to maximizing this entropy. Recall the von Neumann entropy expressed as a function of the density matrix,

$$S_{vN} = -\operatorname{Tr}(\rho \ln \rho) \tag{6.2}$$

  Going from pure-state to mixed-state increases S_{vN}. It is tempting to relate the maximizing of the tunneling probability with the modified wavefunction to the maximizing of the von Neumann entropy S_{vN} for the tunneling process.

- It is possible that such a SOUP cannot be derived from string theory as we know it, since the derivation of the wavefunction requires a knowledge of the full wavefunction $\Psi(a=0, x_n)$ of nothing (i.e., no classical spacetime). This means that SOUP must be postulated. If so, it is even more important to take a phenomenological approach to learn about SOUP. In this case, it is unclear how to differentiate SOUP from the anthropic principle. One may view this as an attempt to quantify the anthropic principle. In this paper, we have taken a more optimistic view. Since classical spacetime are merely derived quantities in string theory, as opposed to the case in general relativity, $\Psi(a=0, x_n)$ should be a meaningful quantity in string theory, although a non-perturbative description may be involved.

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A. Tunneling amplitude

The Feynman functional integral is over all 4-geometries with a spacelike boundary on which the induced metric is h_{ij} and which to the past of that surface there is nothing (see figure 1). $\Psi[h_{ij}]$ satisfies the Wheeler-DeWitt equation:

$$\mathcal{H}\Psi = \left(-G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - {}^3R(h)h^{1/2} + 2\Lambda h^{1/2}\right)\Psi[h_{ij}] = 0 \tag{A.1}$$

where G_{ijkl} is the metric in superspace:

$$G_{ijkl} = \frac{1}{2h^{1/2}}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}) \tag{A.2}$$

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where ${}^3R$ is the scalar curvature of the intrinsic geometry of the 3-surface. For the de Sitter metric, $\mathcal{H}$ reduces to, up to a constant factor,

$$\mathcal{H} = \frac{1}{2}\left(-\frac{p^2}{a} - a + H^2a^3\right) \tag{A.3}$$

where $p = \dot{a}a$. Upon quantization, $p \to id/da$, and the Wheeler-DeWitt equation becomes

$$\left(\frac{-d^2}{da^2} + U(a)\right)\Psi(a) = 0 \qquad U(a) = a^2(1 - H^2a^2) \tag{A.4}$$

which takes the form of a 1-dimensional Schroedinger equation for a particle described by the coordinate $a(t)$. This is illustrated in figure 6. Adding matter fields χ gives the equation:

$$\left(\frac{-\partial^2}{\partial a^2} + U(a) + \frac{\partial^2\chi}{\partial\chi^2} - \chi^2 - 2\epsilon_o\right)\Psi(a,\chi) = 0 \tag{A.5}$$

where ϵ_o comes from the possibility of a matter-energy renormalization. Let

$$\Psi(a,\chi) = \Sigma_n\psi_n(a)u_n(\chi) \tag{A.6}$$

then the above equation becomes two equations:

$$\begin{aligned} \frac{1}{2}\left(\frac{-d^2}{d\chi^2} + \chi^2\right)u_n(\chi) &= \left(n + \frac{1}{2}\right)u_n(\chi) \\ \frac{1}{2}\left(\frac{-d^2\psi_n}{da^2} + U(a)\psi_n\right) &= \epsilon\psi_n \end{aligned} \tag{A.7}$$

where $\epsilon = n + 1/2 - \epsilon_0$. For $\epsilon \to 0$ the above equation for ψ_0 just becomes eq. (A.4) which describes the ground state of the universe. For nonzero ϵ one obtains the excited states of the universe.

Ignoring the matter fields, this reduces to eq. (A.4). The classically allowed region is $a \geq 1/H$, and its solutions are:

$$\Psi_\pm(a) \simeq e^{\pm i\int_{1/H}^{a} p(a')da' \mp i\pi/4}\,. \tag{A.8}$$

In the classically forbidden region, i.e., the under-barrier $a < 1/H$ region, the solutions are

$$\tilde{\Psi}_\pm(a) \simeq e^{\pm\int_a^{1/H}|p(a')|da'}\,. \tag{A.9}$$

The under-barrier wavefunction is a linear combination of $\tilde{\Psi}_+(a)$ and $\tilde{\Psi}_-(a)$, where $\tilde{\Psi}_+(a)$ decreases exponentially while $\tilde{\Psi}_-(a)$ grows exponentially with increasing a. There are a number of proposals in obtaining the tunneling probability from this wavefunction. For quantum tunneling, the Hartle-Hawking wavefunction is given by

$$\Psi[h_{ij}] = \int_\emptyset^{h_{ij}}[dg]e^{-S_E[g]} \simeq e^{-S_E} \tag{A.10}$$

where S_E is the euclidean action of the corresponding instanton solution. This corresponds to the simple extension from quantum field theory. However, in gravity, S_E is unbounded

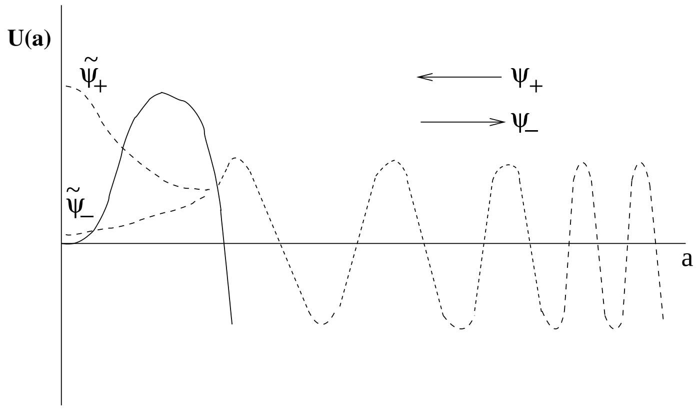

Figure 6: The tunneling wavefunction. The growing and the decaying components under the barrier combine to form the outgoing and incoming waves on the right. If only the growing mode is present (the Hartle-Hawking wavefunction), or if only the decaying mode is present (the Linde wavefunction), there will be both outgoing and incoming waves on the right. Vilenkin demands a particular combination so there is only outgoing wave. Decoherence will yield the outgoing wave as the classical solution.

from below. In fact, for the de Sitter instanton solution, one has eq. (1.1). In terms of the under-barrier wavefunction, this corresponds to the dominance of the growing $\tilde{\Psi}_-(a)$. Our work is built on this interpretation.

According to DeWitt [74], the wavefunction at $a = 0$ should vanish: $\Psi_{DW}(a = 0) = 0$. DeWitt believes that $a = 0$ corresponds to a cosmological singularity, so his condition avoids the occurrence of the singularity. This implies that $\tilde{\Psi}_+(a)$ and $\tilde{\Psi}_-(a)$ cancels each other at $a = 0$. Since they are comparable at $a = 0$, $\tilde{\Psi}_-(a)$ dominates around $a = 1/H$. This gives the same result as the Hartle-Hawking wavefunction. This same result is also obtained if one assumes a generic wavefunction around $a = 0$, so that the 2 wavefunctions $\tilde{\Psi}_+(a)$ and $\tilde{\Psi}_-(a)$ are comparable around $a = 0$.

If one assumes that only the decaying wavefunction is present at $a \sim 0$, then one obtains the following wavefunction, $\Psi_L(a) \simeq e^{+S_E}$, as proposed by Linde [33]. According to Vilenkin [34, 75], $\Psi_-(a)$ corresponds to an expanding universe while $\Psi_+(a)$ corresponds to a contracting universe. Suppose one wants only an expanding universe in the classically allowed region $a > 1/H$, that is, $\Psi(a > 1/H) = \Psi_-(a)$. Then $\tilde{\Psi}_+(a)$ and $\tilde{\Psi}_-(a)$ should match around $a = 1/H$, or

$$\Psi\left(a < \frac{1}{H}\right) = \tilde{\Psi}_+(a) - \frac{i}{2}\tilde{\Psi}_-(a)\,. \tag{A.11}$$

In this case, the under-barrier region is dominated by $\tilde{\Psi}_+(a)$ and the tunneling wavefunction is $\Psi_T \sim e^{-3\pi/2G_N\Lambda}$. Although this agrees with Ψ_L for the de Sitter instanton, it is different in that

$$\Psi_T = \int [dg] e^{iS} \to e^{-|S_E|} \tag{A.12}$$

so for ordinary quantum field theories with bounded euclidean action, it reduces to the answer we know. This alternative possibility was also mentioned in ref. [17]. However, as pointed out by Rubakov [76] and others, the arrow of time is not predetermined since there is no outside observer. Furthermore, the introduction of scalar fields into the gravity theory will lead to catastrophic radiation of scalar fields (due to the sign flip of the matter field effective action). For our purpose, this proposal will lead to the breakdown of either the semiclassical approximation or to the unboundedness of the string vacuum. As pointed out in Ref [31], decoherence will select the growing mode as the classical evolution of the universe.

The Hartle-Hawking wavefunction implies that quantum tunneling prefers a de Sitter universe with size $a \simeq 1/\sqrt{\Lambda} \to \infty$, which is clearly unphysical. This is a major reason in the search of an alternative wavefunction. Here we shall start with the Hartle-Hawking wavefunction. As discussed in the text, this wavefunction needs an important improvement due to the coupling/presence of other modes/fields.

B. Feynman paths with complex metric

Here we follow closely the formalism of Halliwell and Louko [29]. Their normalization is slightly different from that used in the text. The metric ansatz is

$$ds^2 = \sigma^2 \left(N(\tau)^2 d\tau^2 + b(\tau)^2 d\Omega_3^2 \right) , \tag{B.1}$$

where $\sigma^2 = 2G/3\pi$. We choose τ to be real and a (piece-wise) constant lapse function: $\dot{N} = 0$.

The Einstein-Hilbert contribution to the action (2.1) is

$$\frac{1}{16\pi G} \int_M d^4x \sqrt{|g|} \, (R - 2\Lambda) = \frac{1}{2} \int d\tau N(-\frac{b\dot{b}^2}{N} + \lambda b^3 - b) + \frac{1}{2} \frac{b^2 \dot{b}}{N} \Big|_{\tau=1} , \tag{B.2}$$

where $\lambda = \sigma^2 \Lambda/3 = \sigma^2 H^2$ and the last term above comes from partial integration of a second derivative term in the Ricci scalar, containing $\ddot{b}$. The extrinsic curvature is

$$K = -\frac{3\dot{b}}{\sigma N b} . \tag{B.3}$$

so the boundary term contribution to the action (2.1) is

$$\frac{-1}{8\pi G} \int_{\partial M} d^3x \sqrt{|h|} K = -\frac{1}{2} \frac{b^2 \dot{b}}{N} |_{\tau=1} . \tag{B.4}$$

and

$$S = \frac{1}{2} \int_0^1 d\tau N(-\frac{b\dot{b}^2}{N} + \lambda b^3 - b) . \tag{B.5}$$

Taking the initial value of $b(0) = 0$ and the final value $b(1) = a$, the solution of the action is

$$b(\tau) = r \sin \left(\frac{N\tau}{r} \right) , \tag{B.6}$$

where $r = 1/\sqrt{\lambda}$. However, we shall leave r unfixed, since we are interested in summing over four-spheres of arbitrary radius r. Instead of r, one may trade it for z, where

$$z \equiv 1 + \frac{\dot{b}(1)}{N} = 1 + \cos\left(\frac{N}{r}\right) \tag{B.7}$$

and

$$r^2 = \frac{a^2}{z(2-z)} \,. \tag{B.8}$$

Inserting this ansatz in the action (B.5) and performing the the τ integral we find

$$S(z,a,T) = S(z,a) = \frac{a^2}{6}\left(1 - z + \frac{\lambda a^2 - 4}{z} + \frac{\lambda a^2}{z^2}\right), \tag{B.9}$$

and the path integral is just a single ordinary integral over z:

$$\Psi(a) = \int_C dwdz\mu(z,a,w)\exp\left(-S(z,a,w)\right). \tag{B.10}$$

Note that we have introduced w to label the paths that have the same degenerate action (B.9), a fact that will be important later.

We shall adopt the steepest-descent method, where the steepest-descent paths are the paths with $Im(S)$ =constant. For $\lambda a_1^2 < 1$, the saddle points are

$$z = 1 \pm (1 - \lambda a_1^2)^{\frac{1}{2}} \,, \tag{B.11}$$

which correspond to $r = \lambda^{-1/2}$ and represent, respectively, less than or more than half a four-sphere with three-boundary with radius a. (We ignore another saddle point at $z = -2$, which apparently corresponds to negative Λ.) The action at these two saddle points has the values

$$S(a_1) = -\frac{1}{3\lambda}\left(1 \mp (1 - \lambda a_1^2)^{\frac{3}{2}}\right). \tag{B.12}$$

respectively. One may envision that N stays real as the tunneling from $b(0) = 0$ to $b(1) = a_1$, at which point the universe starts evolving classically in an inflationary epoch, where a grow from a_1 to $\hat{a}$. That is, at $\tau = 1$, we switch from the euclidean metric (for S^4) to a lorentzian metric (for de Sitter space), i.e., the lapse function N goes from a real constant to a pure imaginary constant. To describe the tunneling of an inflationary universe from nothing, we must deal with values of N in the complex N plane.

Once we are ready to entertain complex N, let us consider the the tunneling from nothing directly to an universe with size $\hat{a}$, where $\lambda\hat{a}^2 > 1$. In this case, the path integral has the saddle points at (again ignoring the saddle point $z = -2$)

$$z = 1 \pm i(\lambda\hat{a}^2 - 1)^{\frac{1}{2}} \,, \tag{B.13}$$

at which the action has the value

$$S(\hat{a}) = -\frac{1}{3\lambda}\left(1 \mp i(\lambda\hat{a}^2 - 1)^{\frac{3}{2}}\right). \tag{B.14}$$

respectively. Here, the saddle points still correspond to $r = \lambda^{-1/2}$, but $r < \hat{a}$. We still have $b(0) = 0$, $b(1) = \hat{a}$ and $\dot{b}(0)/N = 1$, but

$$\begin{aligned}
N &= r\left[\frac{\pi}{2} \mp iu\right] \\
a(\tau) &= r \sin\left(\frac{\pi\tau}{2}\right) \cosh(u\tau) \mp i \cos\left(\frac{\pi\tau}{2}\right) \sinh(u\tau) \\
\cosh(u) &= \lambda^{1/2}\hat{a}\,.
\end{aligned} \tag{B.15}$$

These new solutions arise only when one performs the analysis with complex metric.

Now, let us come to the issue of degeneracy. Introducing $T = N\tau$, the action (B.5) becomes

$$S = \frac{1}{2}\int_0^N dT \left[-b\left(\frac{db}{dT}\right)^2 + \lambda b^3 - b\right] \tag{B.16}$$

this is a complex integral over T of an analytic function in which the contour in the complex T plane can take a variety of paths, a subset of which is shown in figure 5.

It is instructive to calculate the extrinsic curvature. Using eqs. (B.3) and (B.6) we find

$$K = -\frac{3\sqrt{\lambda}}{\sigma}\,\mathrm{cotan}\left(\tau\left(\frac{\pi}{2} \mp iu\right)\right). \tag{B.17}$$

For τ in the interval $0 \le \tau \le 1$, K is in general a complex number. At $\tau = 1$ we find

$$K = -\frac{3i}{\sigma\hat{a}}(\lambda\hat{a}^2 - 1)^{\frac{1}{2}}\,. \tag{B.18}$$

C. Decoherence

Decoherence introduced in eq. (1.2) is brought about by lifting the degeneracy of complex metric paths by introducing matter degrees of freedom. However, the concept of decoherence is relevant and applicable in a broader framework. Decoherence can be understood in very general terms in quantum mechanics as arising due to the interaction between the system and the environment. The effect of decoherence has been explored in the literature in various quantum mechanical situations, including quantum cosmology. In any quantum mechanical problem there is a system, consisting of the part that we are interested in studying, and the environment, that consists of the part that is not directly relevant. The environment can interact with the system in such a way that leads to a loss of quantum mechanical features (i.e., a loss of coherence) and this process is called decoherence. This may be compared to the quantum Zeno effect where a continuous measurement of the system by the environment leads to the suppression of its tunneling [77, 78].

A simple example consists of an electron beam (the system) in the double slit experiment in quantum mechanics with a gas of molecules between the slits and the screen (the environment). The random interactions between the gas molecules and the electrons can lead to a loss of coherence and the interference pattern can get washed out leading to a decoherent pattern on the screen. The idea of decoherence has also been applied to quantum cosmology to study the semi-classical evolution of the universe. Although one would expect the universe to evolve as some superposition of various possible histories (for example, a quantum mechanical superposition of the growing and the decaying mode in the case of the de Sitter solution), decoherence allows the universe to evolve classically.

The decoherence term in eq. (1.2) ought to arise as a result of such interaction between the system (the gravitational instanton) and the environment (consisting of all other fields) [30, 31, 32]. Decoherence should forbid the quantum tunneling of a large (macroscopic) sized universe. Whereas in previous works [30, 31] the idea of decoherence has been applied to the outside barrier region of the potential in eq. (A.4) to study the classical evolution of the universe, we attribute the origin of the decoherence term in eq. (1.2) to the decoherence mechanism in the under barrier region. So it is an extension of the previous ideas to the under barrier region.

It is convenient to use the language of density matrix to study decoherence. It is also possible to cast the arguments using path integral language but we shall use the density matrix language here. Consider the total wavefunction $|\Psi\rangle$ of the form:

$$|\Psi\rangle = \sum_n c_n |S_n\rangle |E_n\rangle \,. \tag{C.1}$$

The corresponding pure-state density matrix is:

$$\rho = |\Psi\rangle\langle\Psi| = \sum_{mn} c_n c_m^* |S_n\rangle |E_n\rangle\langle E_m|\langle S_m| \tag{C.2}$$

where $|S_n\rangle$ are states of the system and $|E_n\rangle$ are states of the environment. In this state, the system and the environment are correlated with each other. We are, however, only interested in the state of the system, not the environment. So the environmental degrees of freedom can be traced over in any calculation of interest. The object of interest is then the reduced density matrix given by:

$$\rho_S = \mathrm{tr}_E\, |\Psi\rangle\langle\Psi| = \sum_{n,m} c_n c_m^* \langle E_m|E_n\rangle |S_n\rangle\langle S_m| \,. \tag{C.3}$$

If the dot products $\langle E_m|E_n\rangle = \delta_{mn}$, the off-diagonal terms of the density matrix vanish so there are no interference terms, and the system displays a pure classical behavior. The reduced density matrix then becomes diagonal:

$$\rho_S = \sum_n |c_n|^2 |S_n\rangle\langle S_n| \,. \tag{C.4}$$

In general, $\langle E_m|E_n\rangle$ is only suppressed when $m \neq n$. Decoherence is the suppression of the off-diagonal terms, so the system behaves almost classically. In quantum cosmology the total wavefunction is

$$\Psi(a, x_n) = \psi_o(a) \prod_{n>0} \chi_n(a, x_n) \tag{C.5}$$

where $\psi_o(a)$ is the Hartle-Hawking wavefunction, and $\chi_n(a, x_n)$ are the contributions from all other fields with amplitudes x_n. These fields interact with the gravitational contribution encoded in $\psi_o(a)$ and the effect of this interaction can be seen by considering the reduced density matrix:

$$\rho(a;a') = \mathrm{tr}_{x_n}\left(|\Psi\rangle\langle\Psi|\right) = \psi_o(a)\psi_o^*(a')\prod_{n>0}\int_{-\infty}^{\infty} dx_n\chi_n(a,x_n)\chi_n^*(a',x_n)\,. \tag{C.6}$$

Here $\rho(a;a')$ should be compared with the density matrix element ρ_{mn}. In the absence of the environmental degrees of freedom, the density matrix element corresponding to the tunneling of the universe as derived from the Hartle-Hawking wavefunction is given by $\rho(a = H^{-1};0) = e^{-S_E}$. However, tracing out the environment is expected to give the decoherence term leading to the wavefunction proposed in section 2,

$$\Psi(a) = \rho(a = H^{-1};0) \simeq e^{-S_E-\mathcal{D}}\,. \tag{C.7}$$

The exact form of $\mathcal{D}$ depends on the particular model that we consider. A calculation can be performed along the lines in Refs [30, 31] with the environment given by the higher multipoles of the matter and geometry as obtained by Halliwell and Hawking [79]. That tracing out these multipoles leads to decoherence in the lorentzian evolution of the universe has been shown in [30]. We are proposing that a similar procedure can be applied to the under barrier euclidean/complex metric regime and that can lead to the appearance of the decoherence term $\mathcal{D}$. In this sense, the inclusion of the complex metric paths and the S'-brane is related to the appearance of decoherence by tracing out the environment. The decoherence should appear in string theory by tracing over the closed and open string degrees of freedom. Hopefully, string theory provides a natural cut-off that is absent in the analysis so far.

Although the exact form of $\mathcal{D}$ is expected to be obtainable only from a complete analysis in string theory, we shall give a heuristic derivation here which shall justify the form of $\mathcal{D}$ we have claimed. Consider a geometry of the type $S^4 \times M$ where M is a 6-dimensional compactified space of volume V_6. Consider only the tensor modes as the environmental degrees of freedom as in [30]. To trace over the environment, we need to know the wavefunction of these tensor modes in the under-the-barrier euclidean region. However, as shown in ref. [79], the form of the tensor mode wavefunctions is the same both for the euclidean and the lorentzian regions and this is especially true for large-N (small wavelengths) modes. In particular, for both the euclidean and the lorentzian cases, the tensor modes are given by:

$$\chi_n(a,x_n) = \left(\frac{na^2}{\pi}\right)^{1/4}\exp\left(-2i\frac{\partial S}{\partial\alpha}x_n^2 - \frac{1}{2}ne^{2\alpha}x_n^2\right) \tag{C.8}$$

where S is the Hamilton-Jacobi parameter. The phase parts differ for the euclidean and the lorentzian regions. But it is the amplitude part of the tensor modes that are important for determining the decoherence term and they are the same, viz. $(na^2/\pi)^{1/4}\exp(-\frac{1}{2}ne^{2\alpha}x_n^2)$ for both regions. Tracing over the tensor modes involves finding

$$\int dx_n\chi_n^*(a,x_n)\chi_n(a',x_n) = \sqrt{2}\left(\frac{a^2+a'^2}{aa'}\right)^{-1/2}. \tag{C.9}$$

This leads to the result we have used in eq. (2.18)

$$\rho_S(a;a') \propto \prod_{n>0} \exp\left(-\frac{(a+a')^2(a-a')^2}{4a^2a'^2}\right) \sim \exp\left(-N\frac{(a+a')^2(a-a')^2}{4a^2a'^2}\right). \tag{C.10}$$

Tracing over only long wavelength modes, the number of such modes is proportional to the spatial volume. So it is only natural that N should be proportional to the 9-volume V_9. Writing V_9 as $V_6\bar{a}^3$, we have argued in section 2 that $\bar{a}^3 = aa'^2$ for consistent decoherence to take place. Any other choice would lead to either zero or no decoherence Here we are staying in the euclidean region and starting with a finite a' and a; we then take the limit where $a' \to 0$. Decoherence is seen if we choose the cut-off scheme $N \propto V_6aa'^2$. Strictly speaking, the whole calculation in quantum gravity is ill-defined at $a' = 0$ where classical space-time does not exist. Another way to see the difficulty is that, close to $a' = 0$, the separation between lower and higher multipoles becomes less and less accurate. The entire perturbative calculation done in ref. [79] breaks down at $a' = 0$. Since classical spacetime are merely derived (i.e., not fundamental) quantities in string theory, the $a' = 0$ limit should be meaningful and hopefully calculable. There have been discussions about the calculation of the wavefunction of the universe in a string theory context in [80].

Perhaps we can convince the reader that the decoherence term $\mathcal{D}\propto V_9$ by considering the case where the universe starts off at some finite a' with energy less than the height of the barrier as described by a nonvanishing ϵ parameter in eq. (A.7). After tunnelling, it becomes a deSitter state on the other side of the barrier in figure 6. The calculation of the decoherence term can now be done with tensor modes and the final result should have $\mathcal{D}\propto V_9$. We expect the qualitative result $\mathcal{D} \propto V_9$ to be true for this analysis. One can then take the limit $\epsilon \to 0$ and obtain the result for the tunneling from nothing. A macroscopic universe would not tunnel because of decoherence.

D. S^{10} with dilaton

In this section we investigate the impact of dilaton on the action for S^{10}. The action with dilaton included in string frame is

$$S = \frac{1}{16\pi G}\int d^{10}x\sqrt{|g|}e^{-2\phi}\left(R - 2\Lambda\, e^{\phi} + 4(\nabla\phi)^2\right). \tag{D.1}$$

After the Weyl transformation $g_{mn} \to e^{\frac{\phi}{2}} g_{mn}$, the equations in Einstein frame are

$$36\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) = -\Lambda e^{\frac{3\phi}{2}} + \frac{1}{4}\dot{\phi}^2 \tag{D.2}$$

$$8\frac{\ddot{a}}{a} + 28\left(\frac{\dot{a}^2}{a^2} - \frac{1}{a^2}\right) = -\Lambda e^{\frac{3\phi}{2}} - \frac{1}{4}\dot{\phi}^2 \tag{D.3}$$

$$\ddot{\phi} + 9\frac{\dot{a}}{a}\dot{\phi} = 3\Lambda e^{\frac{3\phi}{2}}\,. \tag{D.4}$$

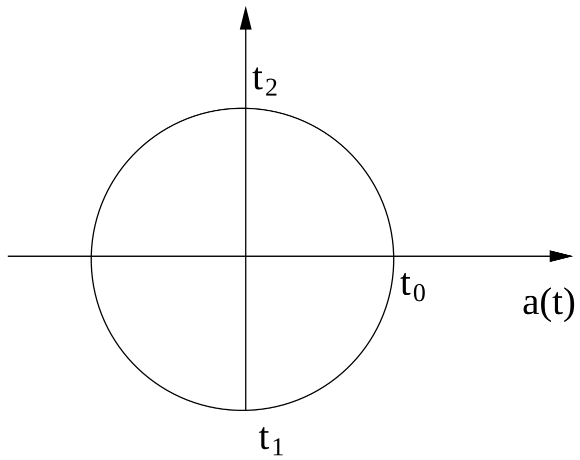

Figure 7: S^4 instanton.

We are interested in the instanton solution with the following properties:

$$\begin{aligned} a(t_1) = a(t_2) &= 0 \\ a(t) &\leq a(t_0) \\ \dot{a}|_{t_0} = \dot{\phi}|_{t_0} &= 0\,. \end{aligned} \tag{D.5}$$

The first two conditions correspond to having a closed instanton with spherical topology. The third condition comes from the analyticity and real-valued properties of the solution in transition from euclidean space-time to the lorentzian space-time (see figure 7). We show that, with the dilaton included, there are apparent singularities at $t = t_1$ and $t = t_2$. In other words, $t_2(t_1)$ corresponds to big-bang (big-crunch).

To begin the proof, suppose the solution is non-singular. The point of special care are t_1 and t_2, where $a(t) = 0$. We focus our investigations near $t = t_2$. The same discussion applies equally to the t_1 case. To have a regular solution, all scalars like R, $R_{mn}R^{mn}$, $R_{mnpq}R^{mnpq}, \ldots$ constructed from the metric components and their derivatives must be regular. In particular, we have

$$R_{mnpq}R^{mnpq} = 36\left[\left(\frac{\ddot{a}}{a}\right)^2 + 4\left(\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2}\right)^2\right]. \tag{D.6}$$

Clearly, $R_{mnpq}R^{mnpq}$ is regular if both $(\ddot{a}/a)$ and $(\dot{a}/a)^2 - 1/a^2$ are regular. Since $a(t)$ vanishes at t_2 by construction, $\dot{a}/a$ will diverge such that $(\dot{a}/a)^2 - 1/a^2$ is finite. Suppose

$$a(t) \sim \alpha[(t - t_2)^p + \beta(t - t_2)^q]\,, \qquad 0 < p < q\,. \tag{D.7}$$

We find

$$\begin{aligned} \frac{\dot{a}}{a} &\sim p(t - t_2)^{-1} + \beta(q - p)(t - t_2)^{q-p-1} \\ \frac{\ddot{a}}{a} &\sim p(p - 1)(t - t_2)^{-2} + \beta\left[q(q - 1) - p(p - 1)\right](t - t_2)^{q-p-2}\,. \end{aligned} \tag{D.8}$$

To have $(\dot{a}/a)^2 - 1/a^2$ and $\ddot{a}/a$ regular, we need $\alpha = -1$ and $q - p \geq 2$, respectively. This implies that $q \geq 3$. So

$$a(t) \sim (t_2 - t) + \beta(t - t_2)^q\,, \qquad q \geq 3\,. \tag{D.9}$$

On the other hand

$$\dot{\phi}^2 = 16\left((\frac{\dot{a}}{a})^2 - \frac{1}{a^2} - \frac{\ddot{a}}{a}\right). \tag{D.10}$$

Near t_2, by using eq.(D.9), we find

$$\dot{\phi}^2 \sim 8\beta q(3-q)(t-t_2)^{q-3}\,. \tag{D.11}$$

Previously we concluded that $q \geq 3$, which indicates that $\dot{\phi} \to 0$ with some positive power.

Now consider the scalar field equation. It is more instructive to write eq.(D.4) in the following form

$$\frac{d}{dt}(a^9\dot{\phi}) = 3\Lambda a^9 e^{\frac{3\phi}{2}}\,. \tag{D.12}$$

Define $F(t) \equiv a^9\dot{\phi}$. At $t = 0$, we demand $\dot{\phi} = 0$, so $F(0) = 0$. Near $t = t_2$, the regularity assumption of the solution requires that $a(t) \sim (t_2 - t), \dot{\phi} \to 0$, which implies that $F(t_2) = 0$ as well. With the conditions $F(0) = F(t_2) = 0$, the mean value theorem of analysis implies that $\dot{F}$ vanishes at some point in the interval $[0, t_2]$. But $\dot{F} = 3\Lambda a^9 e^{\frac{3\phi}{2}}$ is always positive definite. We conclude that regularity at $t = t_2$ is a false assumption and the system develops a singularity at $t = t_2$. It is easy to see that the only property of the dilaton exponential potential which was used to verify the existence of the singularity is the fact that $\partial V/\partial\phi > 0$. This means that the singularity is a generic properties of any scalar field with monotonic potential.

In order to evaluate the action we need to find the form of the singularity. Using eq. (D.7), from eq. (D.10) we find

$$\dot{\phi}^2 \sim 16\left(\frac{p}{(t-t_2)^2} - \frac{1}{\alpha^2(t-t_2)^{2p}}\right). \tag{D.13}$$

To have $\dot{\phi}^2 > 0$, one requires $p \leq 1$, which implies that $\dot{\phi} \sim \frac{1}{t-t_2}$ up to a coefficient. The possibility of $p = 1$ is excluded. The previous discussion indicates that $F(t) \equiv a^9\dot{\phi}$ should not vanish at $t = t_2$, which is not possible for $a(t) \sim (t - t_2)$ and $\dot{\phi} \sim \frac{1}{t-t_2}$. This will fix the coefficient of $\dot{\phi}$ and we find $\dot{\phi} = \pm\frac{4\sqrt{p}}{(t-t_2)}$ and $\phi \sim \pm 4\sqrt{p}\,\ln|t - t_2|$. Using the scalar field equation, one can easily see that the positive branch solution for ϕ is not allowed and matching the most singular power of $\frac{1}{(t-t_2)}$ will fix $p = 1/9$. In summary, we find

$$a(t) \sim (t_2 - t)^{\frac{1}{9}}\,, \qquad \phi(t) \sim \frac{4}{3} ln|t - t_2|\,. \tag{D.14}$$

To evaluate the action, from eqs. (D.2) and (D.3), $\dot{\phi}$ and $\Lambda e^{\frac{3\phi}{2}}$ can be expressed in terms of $a(t)$ and its derivatives. We find

$$S = \frac{1}{16\pi G}\int d^{10}x\, a^9\left[2\left(\frac{\ddot{a}}{a}\right) + 16\left(\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{a^2}\right)\right]. \tag{D.15}$$

Near $t = t_2$, the singular terms in the bracket cancel each other and the integrand behaves like $(t_2 - t)^{\frac{7}{9}}$ and the action is finite.

Although the analysis are carried out only for S^{10}, one can expect the same qualitative result apply for other instantons considered in section 3 and the dilaton inclusion will not change the result.

E. $S^4 \times S^6$ and other instantons

We start with the 10-dimensional euclidean metric ansatz for $S^{1+n_1} \times S^{n_2}$ $(n_1+n_2+1 = 10)$ with the metric (3.5). The Einstein equation $G_{\mu\nu} = 8\pi G_{10} T^{vac}_{\mu\nu} = -(\Lambda_a g^{(1+n_1)}_{\mu\nu}, \Lambda_b g^{(n_2)}_{\mu\nu})$ becomes (the time-time component, the space-space component in S^{1+n_1} and the space-space component in the S^{n_2} respectively)

$$
\begin{aligned}
-\frac{n_1(n_1-1)}{2}\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)-\frac{n_2(n_2-1)}{2}\left(\frac{\dot b^2}{b^2}-\frac{1}{b^2}\right)-n_1 n_2\frac{\dot a}{a}\frac{\dot b}{b} &= \Lambda_a \\
-(n_1-1)\frac{\ddot a}{a}-n_2\frac{\ddot b}{b}-\frac{(n_1-1)(n_1-2)}{2}\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)- & \\
-\frac{n_2(n_2-1)}{2}\left(\frac{\dot b^2}{b^2}-\frac{1}{b^2}\right)-(n_1-1)n_2\frac{\dot a}{a}\frac{\dot b}{b} &= \Lambda_a \\
-n_1\frac{\ddot a}{a}-(n_2-1)\frac{\ddot b}{b}-\frac{n_1(n_1-1)}{2}\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)- & \\
-\frac{(n_2-1)(n_2-2)}{2}\left(\frac{\dot b^2}{b^2}-\frac{1}{b^2}\right)-n_1(n_2-1)\frac{\dot a}{a}\frac{\dot b}{b} &= \Lambda_b
\end{aligned}
\tag{E.1}
$$

where the dot is with respect to euclidean time τ. (Different Λs may be realized by having some $D9-\bar D9$ branes plus, for example, $D(1+n_1)-\bar D(1+n_1)$ pairs of branes behaving like dust in the extra n_2 dimensions.) The instanton solution corresponds to a S^{1+n_1} bounce with a static S^{n_2} (constant $b(\tau) = b$):

$$
\begin{aligned}
-\dot a^2+1 &= H^2 a^2 \\
a(\tau) &= H^{-1}\cos(H\tau) \\
H^2 &= \frac{2(n_1-1)\Lambda_a - n_2(n_1-1)((\Lambda_a-\Lambda_b)}{n_1(n_1-1)(n_1+n_2-1)} \\
b^2 &= \frac{(n_2-1)(n_1+n_2-1)}{(\Lambda_a+\Lambda_b)+n_1(\Lambda_a-\Lambda_b)}\,.
\end{aligned}
\tag{E.2}
$$

Here S^{n_2} is cosmologically stabilized, with radius b. For $\Lambda_a = \Lambda_b = \Lambda$, the above result reduces to

$$
\begin{aligned}
b^2 &= \frac{(n_2-1)(n_1+n_2-1)}{2\Lambda} \\
H^2 &= \frac{2\Lambda}{n_1(n_1+n_2-1)}\,.
\end{aligned}
\tag{E.3}
$$

For the case $n_2 = 0$, the last component of the above Einstein equation is absent and we have $H^2 = 2\Lambda/n(n-1)$. For S^{10}, this becomes $H^2 = \Lambda/36$.

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For the case of $S^4 \times S^6$ with equal Λ, $n_1 = 3$ and $n_2 = 6$, and we get $b = \sqrt{20/\Lambda}$ and $H^2 = \Lambda/12$. By Wick rotating the time axis to Minkowski metric: $\tau \to it$, we have

$$a(t) = H^{-1}\cosh(Ht) \tag{E.4}$$

this is just the usual $1+3$ dimensional deSitter space-time with a static S^6 making up the extra dimensions. It describes a universe that is contracting at $t < 0$, reaches its minimum size ($a_{\text{minimum}} = 1/H$) at $t = 0$ and then expands for $t > 0$. For large t, the expansion is exponentially, corresponding to an inflationary universe. Since the cosmological constant is due to brane-anti-brane pairs, this universe is always classically unstable. The presence of tachyonic modes will lead to their eventual annihilations, which will end inflation. On the other hand, the above euclidean solution describes a compact space; it is defined only for $|\tau| \leq \pi/2H$. Together, they describe the quantum tunneling process (See figures 1 and 2). Starting from "nothing" (i.e., no space and no time), the instanton solution bounces at the classical turning point $a = 1/H$ (where $\dot{a} = 0$), and continues at Minkowski time t.

The euclidean action S_E and the spatial volume are easy to determine. As an example, consider the case of $S^4 \times S^6$. The geometrical factor $\int \sqrt{g} d^{10}x$ factorizes to $A_4 \times A_6$. The radius of S^4 is just the Hubble radius, H^{-1}. The radius of S^6 is $b = \sqrt{20/\Lambda}$, so $A_4 = \frac{8}{3}\pi^2 H^{-4}$ and $A_6 = \frac{16}{15}\pi^3 b^6$. So

$$\begin{aligned} S_E &= -\frac{1}{16\pi G_{10}} \times \frac{5\Lambda}{2} \times \int \sqrt{g} d^{10}x \\ &= \frac{1}{16\pi G_{10}} \times \frac{5\Lambda}{2} \times A_4 \times A_6 \\ &= \frac{1}{16\pi G_{10}} \times \frac{5\Lambda}{2} \times \frac{8}{3}\pi^2 (H^{-1})^4 \times \frac{16}{15}\pi^3 b^6 \\ &= 1.57187 \times 10^{10}\,. \end{aligned} \tag{E.5}$$

The generalization to $S^{n_1+1} \times S^{n_2} \times \cdots \times S^{n_k}$ instantons is completely straightforward. One finds:

$$\begin{aligned} H^2 &= \frac{2\Lambda}{n_1(D-2)} \\ a_1(t) &= H^{-1}\cos(Ht) \\ a_l &= \sqrt{\frac{(n_l-1)(D-2)}{2\Lambda}} \end{aligned} \tag{E.6}$$

where $l = 2, 3, \ldots, k$ and $D = \sum_{i=1}^{k} n_i + 1$. The cosine dependence of S^{n_1+1} becomes a cosh dependence in lorentzian space-time and the solution describes, after tunneling, inflating $n_1 + 1$ dimensional space-time with $\sum_{i=2}^{k} n_i$ extra dimensions that are static and have the geometry of $S^{n_2} \times \cdots \times S^{n_k}$.

One may also consider instantons with torus fibered over sphere: $S^{n_1+1} \times T^1 S^{n_2} \times T^m$. For example, consider T^1 fibered over S^{n_2} as implied by the notation $\cdots \times T^1 S^{n_2} \times \cdots$.

The metric is given by:

$$
\begin{aligned}
ds^2 =&\ d\tau^2 + a(\tau)^2\left(\frac{dr^2}{1-r^2}+r^2 d\Omega^2_{(n_1-1)}\right)+\\
&+b^2\left(\frac{d\rho^2}{1-\rho^2}+\rho^2 d\Omega^2_{(n_2-1)}\right)+c(\theta)^2 d\alpha^2+d^2\sum_{i=1}^{m} d\beta_i^2
\end{aligned}
\tag{E.7}
$$

where α and β_i are the scale factors of the torii. θ is the azimuthal coordinate of S^{n_2} and the θ dependence of the torus scale factor $c(\theta)$ expresses the fibration of the torus T^1 on S^{n_2}. $c(\theta)$ and d are taken to be time independent as we are looking for solutions where the extra dimensions are time independent. The euclidean Einstein equations for this metric are:

$$
\begin{aligned}
-\frac{1}{2}n_1(n_1-1)\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)-\frac{1}{2}n_2(n_2-1)\frac{1}{b^2}-\frac{1}{b^2}\left(\cot\theta\frac{c'(\theta)}{c(\theta)}+\frac{c''(\theta)}{c(\theta)}\right) &= \Lambda\\
-\frac{\ddot a}{a}+(n_1-1)\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right) &= 0\\
-n_1\frac{\ddot a}{a}-\frac{1}{2}n_1(n_1-1)\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)-\frac{1}{2}(n_2-1)(n_2-2)\frac{1}{b^2}-\frac{1}{b^2}\left(\cot\theta\frac{c'(\theta)}{c(\theta)}\right) &= \Lambda\\
-n_1\frac{\ddot a}{a}-\frac{1}{2}n_1(n_1-1)\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)-\frac{1}{2}(n_2-1)(n_2-2)\frac{1}{b^2}-\frac{1}{b^2}\left(+\frac{c''(\theta)}{c(\theta)}\right) &= \Lambda\\
-n_1\frac{\ddot a}{a}-\frac{1}{2}n_1(n_1-1)\left(\frac{\dot a^2}{a^2}-\frac{1}{a^2}\right)-\frac{1}{2}n_2(n_2-1)\frac{1}{b^2} &= \Lambda\\
\frac{\dot b}{b}\frac{c'}{c}-\frac{1}{c}\frac{\partial^2 c}{\partial\theta\partial t} &= 0\,.
\end{aligned}
\tag{E.8}
$$

The solution of euclidean Einstein equations for this metric is:

$$
\begin{aligned}
a(t) &= H^{-1}\cos(Ht)\,,\\
\frac{1}{b^2} &= \frac{2\Lambda}{[2(n_1+1)+n_2(n_2-1)]}\,,\\
c(\theta) &= \cos(\theta)\,,\\
H^2 &= \frac{2}{n_1 b^2}\,.
\end{aligned}
\tag{E.9}
$$

The static scale factor d of the torus T^m is not fixed by these equations.

One may consider $S^{1+n_1}\times T^1 S^2\times T^m$ type instantons in $10-D$ $(1+n_1+1+2+m=10)$ spacetime and see how the action varies. Using (E.9) one can find the action for such an instanton. The action for an instanton $S^{1+n_1}\times T^1 S^{n_2}\times T^m$ in D-spacetime dimensions is given by:

$$
\begin{aligned}
S_E &= -\frac{1}{16\pi G^D}\times\frac{4\Lambda}{D-2}\times\text{Volume}\\
&= -\frac{4\Lambda}{16\pi G^D(D-2)}\frac{2\pi^{(2+n_1)/2}(H^{-1})^{1+n_1}}{\Gamma(\frac{2+n_1}{2})}\frac{2\pi^{(2+n_2)/2}(b)^{n_2}}{\Gamma(\frac{2+n_2}{2})}\left(\frac{2\pi A}{M_s}\right)^{1+m}
\end{aligned}
\tag{E.10}
$$

where A is the size of the torii in string units. Using (E.9), (3.4), (3.3), for the case $D = 10$ and $n_2 = 2$ one obtains:

$$S_E = -\left(16\pi A^7\right) \frac{1}{n_1 A^{n_1} (n_1/2)!} \left(\frac{\pi}{2} n_1 (2+n_1)\right)^{(3+n_1)/2} . \tag{E.11}$$

F. Locating the preferred inflationary state

Consider the KKLMMT scenario. The Kähler potential is

$$\mathcal{K} = -3\ln[-i(\rho-\bar\rho)] - \ln[-i(\tau-\bar\tau)] - \ln\left(-i\int_{\mathcal{M}} \Omega\wedge\bar\Omega\right). \tag{F.1}$$

The supersymmetric minimum is given by $D_z W = D_\tau W = D_\rho W = 0$ (where $D_i W = \partial_i W + \partial_i \mathcal{K} W$), which lead to, respectively,

$$\frac{M}{2\pi i}(1+\ln z) - \left(K + K'\frac{\partial z'}{\partial z}\right)\tau = 0 \tag{F.2}$$

$$M\mathcal{G} - (Kz + K'z')\bar\tau = -Ae^{ia\rho} \tag{F.3}$$

$$M\mathcal{G} - (Kz + K'z')\tau = \left(\frac{i}{3}a(\rho-\bar\rho) - 1\right) Ae^{ia\rho} . \tag{F.4}$$

Here we present solutions of eqs.(F.2), (F.3) and (F.4). To simplify the analysis we take $\tau = ie^{-\phi} = i/g_s$, $\rho_{\rm min} = i\sigma$ while z and z' are taken to be real. Also we assume $z'(z) \sim \mathcal{O}(1)$ and $\partial z'(z)/\partial z \sim 0$ for small z. In the limit of physical interest where K/g_s is large, we find from eq. (F.2) that

$$z = \exp\left(-\frac{2\pi K}{M g_s}\right). \tag{F.5}$$

In this limit, from the sum and difference of the remaining two equations, respectively, we find

$$a\sigma = -3 - LW(-1,-m) \tag{F.6}$$

$$\frac{1}{g_s} = -\frac{i}{3K'z'(0)} A\, a\sigma \exp(-a\sigma) = \frac{|\mathcal{G}(0)|M}{K'z'(0)}\beta(m) \tag{F.7}$$

where we have introduced $m \equiv 3e^{-3}M|\mathcal{G}(0)A^{-1}|$ and $\beta(m) \equiv \frac{3+LW(-1,-m)}{LW(-1,-m)}$. To have a solution, we need $m < 3e^{-3} \sim 0.15$. The function $y(x) = -LW(-1,-x)$ is the LambertW functions, which is the solution of the equation $ye^{-y} = x, x < e^{-1}$, for $y > 1$. Using eq. (F.7) in z expression, eq. (F.5), we find

$$z = e^{-2\pi|\mathcal{G}(0)|\beta(m)K/z'(0)K'} . \tag{F.8}$$

We are interested in maximizing $\mathcal{F} \equiv -S_E - \hat{c}M_s^9 V_9$ in terms of fluxes, K, M and K'. Using the solutions found for $z, \rho(\sigma)$ and g_s and using eqs. (4.12) and (4.14) for S_E and $cM_s^9 V_9$, respectively, we find

$$\mathcal{F} = \bar{A} f_1(m) e^{\frac{8\pi|\mathcal{G}(0)|K\beta(m)}{3K'}} - \hat{c}\bar{B} f_2(m) e^{\frac{4\pi|\mathcal{G}(0)|K\beta(m)}{K'}} \tag{F.9}$$

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where

$$
\begin{aligned}
\bar{A} &\equiv \frac{12}{(2\pi)^9}\left(\frac{e^3}{3z'(0)}\right)^3 |A|^3 a^{-3} \\
\bar{B} &\equiv \frac{2\sqrt{3}}{(2\pi)^5}\left(\frac{e^3}{3z'(0)}\right)^{\frac{3}{2}} |A|^{\frac{3}{2}} a^{-\frac{15}{4}} \\
f_1(m) &\equiv m^3 \frac{(3+LW(-1,-m))^6}{|LW(-1,-m)|^3} \\
f_2(m) &\equiv m^{\frac{3}{2}} \frac{|3+LW(-1,-m)|^{\frac{21}{4}}}{|LW(-1,-m)|^{\frac{3}{2}}} .
\end{aligned} \tag{F.10}
$$

From the combination of $\frac{\partial}{\partial K}\mathcal{F}=0$ and $\frac{\partial}{\partial M}\mathcal{F}=0$, we obtain

$$
\begin{aligned}
3f_1' f_2 - 2f_1 f_2' &= 0 \\
e^{\frac{pK\beta(m)}{2K'}} &= \frac{2\bar{A}}{3c\bar{B}}\frac{f_1(m)}{f_2(m)} .
\end{aligned} \tag{F.11}
$$

The root of the first equation, independent of a and $|A|$ is $m_{\max} \sim 0.06$. It turns out that the condition $\frac{\partial}{\partial K'}\mathcal{F}=0$ is not quite compatible with $\frac{\partial}{\partial K}\mathcal{F}=0$. On the other hand, K' appears only in the denominator of the exponential terms in $\mathcal{F}$. Since the fluxes are quantized we can choose the smallest possible integer value for K' to maximize $\mathcal{F}$; that is, $K'=1$.

From eqs. (F.11) and (F.11), respectively, we find

$$
\begin{aligned}
M &= \frac{m\,e^3}{3}\left|\frac{A}{\mathcal{G}(0)}\right| \\
K &= \frac{3}{4\pi\mathcal{G}(0)\beta(m)} \ln\left(\frac{4\sqrt{3}\,a^{\frac{3}{4}}}{(2\pi)^4}\frac{f_1(m)}{f_2(m)}\left|\frac{e^3 A}{3z'(0)}\right|^{\frac{3}{2}}\frac{1}{\hat{c}}\right) .
\end{aligned} \tag{F.12}
$$

Correspondingly, the warp factor is given by

$$
\left(\frac{r_0}{R}\right) \sim z^{1/3} = \left(\frac{4\sqrt{3}\,a^{\frac{3}{4}}}{(2\pi)^4}\frac{f_1(m)}{f_2(m)}\left|\frac{e^3 A}{3z'(0)}\right|^{\frac{3}{2}}\frac{1}{\hat{c}}\right)^{-1/2} . \tag{F.13}
$$

From eqs. (4.11), (F.22) and (F.17), we find

$$
\left(\frac{M_{Pl}}{M_s}\right)^2 = \frac{2\,e^6 m^2 \beta(m)^2}{9\,(2\pi)^7 z'(0)} |3+LW(-1,-m)|^{\frac{3}{2}}\, a^{-\frac{3}{2}} |A|^2 . \tag{F.14}
$$

To get a better understanding of the results, we will use the approximate values for $\beta(m)$, $f_1(m)$ and $f_2(m)$ at the maximum point $m_{\max} \sim 0.06$:

$$
\beta(m) \sim 0.3\,, \qquad f_1(m) \sim 10^{-5}\,, \qquad f_2(m) \sim 5\times 10^{-3} . \tag{F.15}
$$

Also the geometric quantities $|\mathcal{G}(0)|$ and $z'(0)$ are basically calculable. We use the GKP approximation and set $|\mathcal{G}(0)| \sim z'(0) \sim 1$, so the expressions for M, K, the warp factor and M_{Pl}/M_s, respectively, simplify to

$$\begin{aligned} M &\sim 0.4 \times |A| \\ K &\sim 0.8 \times \ln\left(10^{-4}\, a^{3/4}\, |A|^{3/2}\, \hat{c}^{-1}\right) \\ \left(\frac{r_0}{R}\right) &\sim 115 \times a^{-3/8}\, |A|^{-3/4}\, \hat{c}^{1/2} \\ \frac{M_{Pl}}{M_s} &\sim 3 \times 10^{-4}\, a^{-3/4}\, |A|\,. \end{aligned} \tag{F.16}$$

The value of function $\mathcal{F}$ at the maximum is

$$\mathcal{F}_{\max} \sim 2 \times 10^{-18}\, a^{-3/2}\, |A|^6\, \hat{c}^{-2}\,. \tag{F.17}$$

Finally, the COBE normalization (that is, the density perturbation $\delta_H \simeq 10^{-5}$) is given in eq.(C.8) of KKLMMT as

$$\delta_H = C_1 N_e^{5/6} \left(\frac{T_3}{M_p}\right)^{1/3} \left(\frac{r_0}{R}\right)^{4/3}, \tag{F.18}$$

where N_e is the number of e-folding and $C_1 \sim 0.4$ in their model. Using eqs. (4.8), (4.11) and (F.17) and taking $N_e \sim 60$, we find

$$\delta_H \sim 2 \times 10^7\, a^{1/2}\, |A|^{-2}\, \hat{c}^{2/3}\,. \tag{F.19}$$

There is an interesting relation between $\mathcal{F}$ and δ_H. One can easily show from eqs. (F.17) and (F.19) that

$$\mathcal{F}_{\max} \sim 1.6 \times 10^4\, \delta_H^{-3}\,. \tag{F.20}$$

For $\delta_H \sim 10^{-5}$, we find

$$\mathcal{F} \sim 10^{18}\,. \tag{F.21}$$

This value for $\mathcal{F}$ is many order of magnitudes bigger than the value of $\mathcal{F}$ for S^{10}.

To check the validity of the low energy supergravity limit, we must verify that $g_s < 1$ and $r \gg 1$(in the unit of M_s). From eqs. (F.7) and (F.17) we find

$$\frac{1}{g_s} = \frac{sM}{2\pi K' z'(0)}\, \beta(m) \sim 0.1 \times |A|\,. \tag{F.22}$$

To satisfy the bound $g_s < 1$, we take $|A| > 10$. From eq. (F.6) we have

$$|\sigma| = r^4 = -\frac{1}{a}\left(3 + LW(-1,-m)\right) \sim \frac{1.2}{a} \tag{F.23}$$

where the approximation $-3 - LW(-1,-m) \sim 1.2$ was used. We see that to have $r \gg 1$, we need $a \ll 1$. For example the value chosen in KKLT for $a = 0.1$, results in $r \sim 2$.

It is easy to generalize the above inflationary scenario to $\mathcal{N}$ pairs of $D3-\bar{D}3$ branes. We also add a very small Λ_{KKLT} like today's cosmological constant as is obtained in KKLT:

$$\Lambda \to \hat{\Lambda} = 8\pi G \mathcal{N} T_3 \left(\frac{r_0}{R}\right)^4 + \Lambda_{\mathrm{KKLT}}\,. \tag{F.24}$$

For $\mathcal{N} \neq 0$, the first term clearly dominates and the second term may be ignored. This corresponds to $\bar{A} \to \mathcal{N}^{-1}\bar{A}$ and $\bar{B} \to \mathcal{N}^{-3/2}\bar{B}$ in eq. (F.9). The extremum can be obtained exactly as before. M is given as in eq. (F.12) while K is given by

$$K \to K + \frac{3}{8\pi \mathcal{G}(0)\beta(m)} \ln \mathcal{N}\,. \tag{F.25}$$

So we see that the flux K is not very sensitive to the value of $\mathcal{N}$.

To compare the inflationary states with the KKLT vacua, we calculate $\mathcal{F}$ for $\mathcal{N}=0$:

$$\mathcal{F}_{KKLT} = \frac{3\pi}{2G_N \Lambda_{KKLT}} \left[1 - \frac{(2\pi)^6}{2\sqrt{3}}\, \hat{c}\, g_s^2 \frac{M_s}{\sqrt{\Lambda_{KKLT}}}\right]. \tag{F.26}$$

To find an estimate for $\mathcal{F}_{KKLT}$, let take $g_s \sim 10^{-1}$, $\hat{c} \sim 10^{-10}$ and $\Lambda_{KKLT}/M_{Pl}^2 \sim 10^{-120}$ corresponding to today's cosmological constant. One can easily verify that the $\mathcal{D}$ term is exponentially larger than $-S_E$, and

$$\mathcal{F}_{KKLT} \sim -10^{172} \left(\frac{M_s}{M_{Pl}}\right). \tag{F.27}$$

This means that the tunneling to a small Λ universe is severely suppressed, in contrast to that suggested by the Hartle-Hawking scenario.

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